

**ESSAYS ON MEASURING INTERNATIONAL AND INTRANATIONAL
BUSINESS CYCLES**

A Dissertation

Presented to

The Faculty of the Department

Of Economics

University of Houston

In Partial Fulfillment

Of the Requirements for the Degree of

Doctor of Philosophy

By

Shushanik Papanyan

May, 2005

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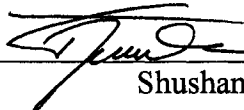
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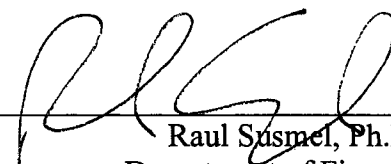

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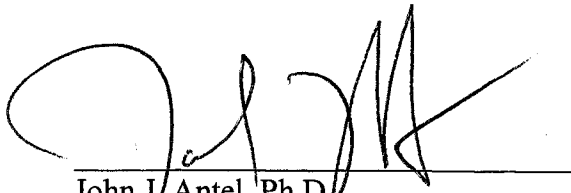
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ABSTRACT

The following studies aim to measure the relative importance of permanent and transitory components of the US business cycles and of the international business cycles of the G7 countries.

Study I: The Relative Importance of Permanent and Transitory Components of Macroeconomic Time Series

The study presents a decomposition of macroeconomic time series such as real GDP, industrial production, consumption, investment, and personal income into permanent and transitory components using highly popular approaches in the recent literature such as the unobserved components model, the Hodrick-Prescott filter, and the Baxter-King filter. It is shown that the weights that these approaches assign to the permanent and transitory shocks in aggregate economic fluctuations are different.

Study II: The Dynamics of Permanent and Transitory Components in International Business Cycles

The study investigates the dynamics of permanent and transitory components that are common across seven developed nations of the world. The common components are modeled to exhibit different behavior in the expansion and recession phases of international business cycles. I employ a multivariate unobserved components model with Markov regime switching. I find that the international business cycle does not exhibit classical contraction and expansion phases. The international permanent component has two phases: a high-growth phase and a low-growth phase, and there is no evidence of an international transitory component. The switch from a high-growth regime to a low-growth regime occurs in the second quarter of 1973. There are no further switches that occur from one regime to another. I also find that Japan is the most sensitive and Germany is the least sensitive to international permanent shocks.

Study III: Driving Forces Behind International Business Cycle Fluctuations: Can One Identify Them?

The study applies cointegrated VAR methodology to identify the long run structure of driving forces behind the international business cycles of the G7 countries. The study shows that the seven countries share four stochastic trends, where the single pushing force of one of the trends is the US. At the same time, the US is not influenced by permanent shocks of the other countries. Shocks to the US are the only ones that have a permanent effect on the France and the UK. The remaining three trends originate as collective stochastic shocks to Canada, France, Germany and Japan. The UK has a transitory effect on the other countries.

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To my niece Mariam
For all the joy she brings to our family
And for her present and future achievements

In memory of my beloved aunt
Eleanora Gharibyan
Whose presence is always missed

Introduction

Empirical literature on the measurement of business cycle fluctuations traces its history back to the first National Bureau of Economic Research (NBER) publications by Burns and Mitchell in 1927, 1938 and 1946. It is particularly influenced by the Burns and Mitchell (1946) book entitled “Measuring Business Cycles,” in which the authors define business cycles as expansions and contractions that occur at the same time in many economic activities and they propose a methodology to measure those economic fluctuations. Furthermore, in their first book published in 1927 they find rough evidence of comovement among different nation’s business cycles. This leads to a definition of international business cycles as the expansions and contractions that occur at the same time in the economies of several countries.

The shortcoming of the existing empirical investigations of national and international business cycles is that most of the methodologies applied are based on the assumption that the investigated time series should be stationary. However, the time series that are commonly used to measure international business cycles usually contain a nonstationary component, so transformation of the series is necessary. To be detrended, the series are either first differenced and or the cyclical component is isolated with the use of HP or BK filters. This poses two problems. First, the interpretation of HP and BK filters depends on the definition of the cyclical component and also is a controversial topic among researchers.¹ Second, detrending ignores the dynamics and importance of the trend in the economic fluctuations.

The objective of subsequent studies is to measure the relative importance of trend in the US business cycles and in the international business cycles among the seven most

¹ The use of those filters for nonstationary series has been criticized by Harvey and Jaeger (1993), Cogley and Nason (1995), and Murray (2003). It has been shown in the literature that detrending nonstationary series using the Hodrick-Prescott and Baxter-King filters can induce spurious cyclical behavior.

developed nations of the world. The above mentioned shortcomings are overcome by adopting the following definition: the trend is the nonstationary component of the series and the cycle is the stationary component of the series. Thus shocks to the trend are permanent and shocks to the cycle are transitory. The trend-cycle decomposition in the studies is conceptually based on Beveridge and Nelson (1981). Beveridge and Nelson (1981) demonstrate that any ARIMA process can be represented as the sum of a random walk and a stationary component. The trend is a random walk with drift and the cycle is the stationary deviation of the series from its random walk trend.

The first study applies the Morley, Nelson, Zivot (2003) unobserved components (UC) decomposition with correlated trend and cycle innovations to four major US macroeconomic variables: real GDP, the index of industrial production, consumption, and personal income. It shows that the trends of those time series account for an overwhelming majority of their observed variance. We also examine the consequences of using the restricted UC model of Harvey (1985) and Clark (1987) and the business cycle filters of Hodrick and Prescott (1997) and Baxter and King (1999). We find that employing these decompositions to integrated time series severely overstates the estimated importance of transitory shocks.

The second study examines the dynamics of trend and cycle that are common to G7 countries. At the same time, the common trend and cycle are allowed to exhibit different behavior depending on whether they are in expansionary or recessionary phases of the international business cycle. The study also provides estimates of the importance of international trend and cycle in the business cycle fluctuations of each nation. It is conducted applying a multivariate UC model with Markov regime switching. I find that the international trend has two phases: a high-growth phase and a low-growth phase and that there is no evidence of an international cycle. The switch from a high-growth regime to a low-growth regime occurs in the second quarter of 1973. I also find that Japan is the most sensitive and Germany is the least sensitive to international permanent shocks.

The last study of the dissertation aims to identify the transmission of permanent shocks from one G7 country to another. This investigation becomes possible with an application of a cointegrated VAR methodology (Juselius, 2005). I find that the shocks

to the US have a permanent influence on the other G7 countries while the US itself is not influenced by the shocks to those countries; the only outside shocks that influence the stochastic trends of Canada, France and the UK are shocks from the US; Germany and Japan transmit permanent shocks between each other and are influenced by shocks from France and the US; and finally, Canada, France, Germany and the US effect Italy's economy, while shocks to Italy do not effect any other country.

By using methodologies that do not require prior detrending of the data, I achieve a deeper understanding of the dynamics of the trend and the relative importance of the trend and cycle in the business cycle fluctuations of the US and of G7 countries.

Study I: The Relative Importance of Permanent and Transitory Components of Macroeconomic Time Series

1. Introduction

The relative importance of permanent and transitory shocks in postwar US output remains an open issue. The interest in understanding the nature of shocks to output has spurred the development of competing econometric tools designed to decompose nonstationary time series into their permanent and transitory components. Some of the most widely used univariate methods are the Beveridge and Nelson (1981) decomposition, the unobserved components models of Harvey (1985) and Clark (1987), the business cycle filters of Hodrick and Prescott (1997) and Baxter and King (1999), as well as linear detrending.² However, it is well documented in the literature that when these alternative decompositions are applied to integrated time series, the relative importance of permanent and transitory components varies drastically.³

Particularly interesting is the disparity in the behavior exhibited by the estimated trend and cycle from the Beveridge-Nelson (BN) decomposition and Harvey-Clark Unobserved Components (UC) models. Both of these decompositions assume a stochastic trend, so that at least some shocks are permanent. However, when one applies the BN and UC decompositions to integrated processes, the former produces a decomposition which indicates that permanent shocks play the largest role, while the latter suggests a smooth trend and a very persistent cycle, much like what is found when either the Hodrick-Prescott or Baxter-King filter is used. How can two approaches that each assumes that the trend component is integrated lead to such drastically different conclusions regarding the relative importance of permanent and transitory components?

² By linearly detrending, we mean taking the residuals from a regression of the log of output on a constant and a linear time trend.

³ See for example the discussions in Watson (1986), Clark (1987), Morley, Nelson, and Zivot (2003), and Murray (2003).

Until recently, this has been somewhat of a conundrum in the applied macro literature, and not much guidance was available in choosing between the BN or UC decomposition.

In a recent paper, Morley, Nelson, and Zivot (2003) demonstrate that the reason for the discrepancy between BN and UC decompositions is that the latter made an unnecessary assumption about the joint behavior of trend and cycle innovations. Harvey (1985) and Clark (1987) assumed that shocks to the trend and cycle are uncorrelated, in order to identify the parameters of the model. Morley, Nelson, Zivot (2003) argue this assumption is in general not necessary for identification. Once the UC model is modified to allow correlated trend and cycle innovations, the UC model coincides exactly with the BN decomposition, and both approaches generate identical trend and cycle estimates.

Given the equivalence of BN and modified UC approaches, as long as one is willing to commit to a particular parameterization, the BN/UC decomposition provides a unified method for decomposing integrated processes into permanent and transitory components.

The question that still remains is, what is the relative importance of permanent and transitory components in macroeconomic time series within this unified framework? Or as Cochrane (1988) put it: “How big is the random walk?” We intend to answer that question in this study. We consider four US macro time series for which there is strong evidence of a unit root: post-war real GDP, the index of industrial production, consumption, and personal income. We decompose these times series into their permanent and transitory components. Our results suggest that the permanent components of these series account for an overwhelming majority of their observed variance.

We also examine the consequences of using inappropriate methods to detrend integrated processes. These include the restricted UC model of Harvey (1985) and Clark (1987), and the business cycle filters of Hodrick and Prescott (1997) and Baxter and King (1999). We find that employing these decompositions to integrated time series drastically overstates the estimated importance of transitory shocks.

The study is organized as follows. Section 2 provides a review of the detrending literature. Section 3 decomposes our times series into permanent and transitory

components using the BN/UC decomposition. Section 4 quantifies the effects of using other detrending methods. Section 5 summarizes and offers concluding remarks.

2. A Review of the Detrending Literature

2.1 The Beveridge-Nelson Decomposition

Beveridge and Nelson (1981) demonstrate that any ARIMA process can be represented as the sum of a random walk and a stationary component. The BN trend is a random walk with drift, where the drift is equal to the growth rate of the underlying series. The BN cycle is therefore the stationary deviation of output from its random walk trend. An implication of the BN decomposition is that the trend and cycle innovations are negatively correlated. Modeling US output as an ARIMA(0,1,1) process, Nelson and Plosser (1982) find that the BN decomposition allocates most of the observed variance in output to the trend component, while the BN cycle small in amplitude and noisy.

2.2 The Harvey-Clark UC Model

Harvey (1985) and Clark (1987) use a UC model to decompose macroeconomic time series into a stochastic trend component and the stationary cyclical component. Let y_t denote the natural log of output. Clark's unobserved components model for output is as follows:

$$y_t = \tau_t + c_t \quad (1)$$

where the trend component, τ_t , contains a unit root:

$$\tau_t = \mu + \tau_{t-1} + \eta_t.^4 \quad (2)$$

The cyclical component, c_t , follows a stationary and invertible ARMA process:

$$\phi_p(L)c_t = \theta_q(L)\varepsilon_t. \quad (3)$$

Furthermore, the innovations to the trend and cycle are assumed to be uncorrelated, *i.e.* $\text{cov}(\eta_t, \varepsilon_t) = \sigma_{\eta\varepsilon} = 0$. Using Nelson and Plosser's real output series, Clark (1987)

⁴ Clark (1987) assumed that the trend growth rate also followed a random walk.

reached a conclusion that was qualitatively different from Nelson and Plosser (1982). He found that most of the variation in economic activity is due to the cyclical component, as the cycle is large in amplitude and highly persistent while trend is very smooth.

Morley, Nelson and Zivot (2003) demonstrate that the reason these two approaches lead to such different conclusions is the assumption of uncorrelated trend and cycle innovations, $\sigma_{\eta\varepsilon} = 0$, is not necessary to identify the parameters of the UC model. Let p and q respectively denote the order of the autoregressive and moving average polynomials in equation (3). Morley, Nelson and Zivot (2003) prove that if $p \geq q + 2$, the parameter $\sigma_{\eta\varepsilon}$ is identified. Once the restriction that $\sigma_{\eta\varepsilon} = 0$ is relaxed, both BN and UC approaches lead to identical trend/cycle decompositions.

2.3 Business Cycle Filters

Implicit in the BN and UC approaches is the definition of the cyclical component as stationary deviations of output from its trend value. Another approach to decomposing output into trend and cycle is the use of business cycle filters, the two most popular due to Hodrick and Prescott (1997) and Baxter and King (1999). These filters are predicated on a definition of the business cycle that relies on frequency components of the data. The cycle is defined as the stationary component of output that remains after a nonstationary series has been passed through an ideal band-pass filter. For the Hodrick-Prescott filter, the frequency band is less than 8 years per cycle, while the Baxter-King band is between 1.5 and 8 years per cycle.

The application of business cycle filters to integrated time series has been criticized in the literature: see Cogley and Nason (1995), Cogley (2001), and Murray (2003). This criticism is predicated on the BN/UC definition of the business cycle as stationary deviations from trend. These papers argue that if one accepts that BN/UC definition of the business cycle, then both the Hodrick-Prescott and Baxter-King filters will induce spurious cyclical behavior if they are applied to integrated time series. As a result, the transitory component is overstated.

In the following pages, we will both measure the relative importance of permanent and transitory components in US macro time series using the BN/UC decomposition, as well as quantify the effects of using inappropriate detrending methods.

3. The Relative Importance of Permanent and Transitory Components

The data we use are US postwar real GDP, the index of industrial production, real personal consumption expenditures, and real disposable personal income. The data are seasonally adjusted. Real GDP, consumption, and personal income are quarterly, and are measured in chained 1996 dollars, from 1947:1 to 2002:2. Industrial production is monthly in chained 1992 dollars from 1947:01 to 2002:06. Each series is in logs and multiplied by 100, so that the cycle can be interpreted as the percentage deviation from trend.

There is strong evidence that each of these time series is integrated. The DF-GLS unit root test of Elliott, Rothenberg, and Stock (1996) with Ng and Perron (2001) MAIC lag selection fails to reject the null of unit root for all four time series. Recognizing the low power of unit roots with such short time series, we conduct Rudebusch's (1993) simulation exercise to assess the power of these unit root tests against the specific trend stationary alternative hypothesis specified by the DF-GLS regression equation. In every case, the power of the unit root tests is at least 80%, which is sufficiently high to support the claim that our series are integrated.

We now proceed to decompose these time series into their permanent and transitory components. Following Morley, Nelson and Zivot (2003), each series is modeled as the sum of a random walk with drift and a stationary and invertible ARMA process. The UC model is characterized by the following equations.

$$y_t = \tau_t + c_t$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t$$

$$\phi_p(L)c_t = \theta_q(L)\varepsilon_t$$

where $\eta_t \sim i.i.d(0, \sigma_\eta^2)$, $\varepsilon_t \sim i.i.d(0, \sigma_\varepsilon^2)$, and $Cov(\eta_t, \varepsilon_t) = \sigma_{\eta\varepsilon}$. Assuming Gaussian errors, this UC model can be cast in state space form, and the maximum likelihood estimates of the parameters, as well as the unobserved trend and cycle, can be computed from the Kalman filter. As Morley, Nelson, and Zivot (2003) demonstrate, when the trend and cycle innovations are allowed to be correlated, the estimated trend from the Kalman filter is numerically identical to the Beveridge-Nelson trend.

To select the order of the ARMA model for the cyclical component, we estimate all possible combinations of p and q , with an upper bound of 2. We then choose the order of the ARMA model based on the Schwarz and Akaike information criteria. Both information criteria choose an AR(2) cycle for real GDP, and AR(1) cycle for personal income and industrial production, and an MA(2) cycle for consumption.

Maximum likelihood estimates are reported in Table 1. All the parameter estimates are significantly different from zero, and standard errors are omitted for brevity. In every case, the standard deviation of the trend innovation is larger than the standard deviation of the cycle innovation. The ratio of innovation standard deviations, $\sigma_\eta / \sigma_\varepsilon$, ranges from 1.16 for consumption to 2.01 for industrial production. This result is consistent with Nelson and Plosser's (1982) finding that trend shocks are relatively more important than shocks to the cycle.

Figure 1 plots the estimated BN/UC trend against the log of the observed series, as well as the estimated cyclical component. The estimated trend is virtually indistinguishable from the underlying series, suggesting the trend explains most of the variance exhibited by the underlying series. The estimated cycle is small in amplitude and noisy.

We now turn to quantifying the relative importance of the trend and cycle based on the BN/UC decomposition. Since the cycle is stationary and the trend is integrated, then as the sample size diverges, the theoretical variance of the trend will also diverge, and the trend will necessarily dominate the cycle. This is necessarily the case irrespective of the particular parameters of the UC decomposition, even for Clark's (1987) model which has a smooth trend and a persistent and high variance cyclical component. To ensure that our measure of trend and cycle variance is not vacuous, we

compute the sample variance of the estimated trend and cycle. For a given sample size, the variance of the trend and cycle are both finite, even though the variance of the former diverges in the limit. We compute the variance of the trend around its drift and initial value. Since the trend is a drifting random walk, it can be written as

$$\tau_t = \mu + \tau_{t-1} + \eta_t = \tau_0 + \mu t + \sum_{j=1}^t \eta_j .$$

We compute the trend variance as the sample size multiplied by the estimated standard deviation of the trend innovation: $T\hat{\sigma}_\eta^2$.

Table 2 reports the sample variances of the estimated trend and cycle components for all four time series. In every case, the trend component dwarfs the cycle and accounts for nearly all of the variance in the observed series. This result corroborates the findings in Beveridge and Nelson (1981), Nelson and Plosser (1982), and Morley, Nelson, and Zivot (2003) that permanent shocks are relatively more important than transitory shocks. Therefore, if one is willing to accept the hypothesis that these series contain a unit root, and the definition of the cyclical component as stationary deviations from trend, then essentially all of the interesting dynamics of these series are captured by the trend component, and the cycle is noisy and largely uninteresting.

4. Comparing BN/UC with Alternative Trend/Cycle Decompositions

In this section, we quantify the effect of using inappropriate detrending methods for integrated time series. We consider the UC model of Harvey (1985) and Clark (1987), and the business cycle filters of Hodrick and Prescott (1997) and Baxter and King (1999).

4.1 An Unobserved Components Model with Orthogonal Trend and Cycle Innovations

In Harvey (1985) and Clark (1987), the trend and cycle innovations are assumed to be orthogonal. We refer to this restricted UC model as UC-0. Morley, Nelson, and Zivot (2003) demonstrate the potential for overstating the importance of cyclical shocks

when the unnecessary assumption of trend and cycle orthogonality is made. We report the maximum likelihood parameter estimates of the UC-0 model in Table 3.

In Table 1, where the trend and cycle are allowed to be freely correlated, in every case the ratio of innovation standard deviations, $\sigma_\eta / \sigma_\varepsilon$, is greater than one. When we restrict the trend and cycle to be uncorrelated, this result is reversed for 3 of the 4 series: for output, consumption, and personal income, the estimated cycle innovation is greater than the trend innovation when we impose the restriction that $\sigma_{\eta\varepsilon} = 0$. This restriction has nontrivial implications for the estimated importance of the trend and cycle. The UC-0 trend and cycle for real GDP are plotted in Figure 2. In contrast to the BN/UC decomposition, the UC-0 model implies a very smooth trend, and a persistent, high amplitude cycle.

Table 4 reports the estimated trend and cycle variances for the UC-0 model. Comparing the estimated trend and cycle from the UC-0 model to those from the unrestricted BN/UC model, we see that when $\sigma_{\eta\varepsilon}$ is assumed to be zero, much more weight is assigned to the stationary cyclical component. For the time series that we consider, the estimated cyclical component is magnified by a factor ranging from 9 to 19! Therefore, the assumption that $\sigma_{\eta\varepsilon} = 0$ essentially induces spurious behavior in the estimated cyclical component.

4.2 Business Cycle Filters

We now consider the effect of using the Hodrick-Prescott and Baxter-King filters when the underlying series is integrated. The application of the HP and BK filters to integrated time series has been criticized by Cogley and Nason (1995) and Murray (2003). These papers argue that if the cycle is defined as stationary deviations of output from its trend value, then HP and BK filters will systematically overstate the importance of the cyclical component when they are applied to processes containing a unit autoregressive root.

For the HP filter, we set the smoothness parameter to 1600 for our quarterly series, and to 14400 for our monthly series. For the BK filter, we use up 12 observations

at the beginning and end of our sample. The HP and BK trend and cycle for real GDP are also plotted in Figure 2. Both filters lead to very similar trend and cycle components. The trend is smooth and slowly evolving, and the cyclical component is a pseudo-periodic process with high amplitude.

The variances of the HP and BK trends and cycles are reported in Table 4. Much like the UC-0 model, these detrending methods overstate the importance of transitory shocks, relative to the BN/UC decomposition. For real GDP, industrial production, and consumption the HP and BK cycles are 6 to 20 times larger than the estimated BN/UC cycle. For personal income, the HP and BK cycles are over 600 times larger than their BN/UC counterpart. These results are consistent with the claims in Cogley and Nason (1995) and Murray (2003) that band-pass filtering integrated time series overstates the importance of the transitory component.

5. Summary and Conclusions

In a recent paper, Morley, Nelson and Zivot (2003) provide a unified framework for decomposing integrated time series into permanent and transitory components. They demonstrate that when the trend and cycle innovations are allowed to be correlated, the Beveridge-Nelson decomposition and unobserved components decompositions coincide. This provides a unified method for decomposing integrated processes into permanent and transitory components.

We use this framework to measure the relative importance of permanent and transitory components in four U.S macroeconomic time series for which there is strong evidence of a unit root. We find that the trend component accounts for nearly all of the variance of these trending series, and that the estimated cyclical component is noisy and small in amplitude. We also quantify the effect of using detrending methods that are inappropriate for integrated time series. We demonstrate that unobserved components models which incorrectly specify orthogonal trend and cycle innovations, and well as the

business cycle filters of Hodrick and Prescott (1997) and Baxter and King (1999),
overstate the importance of transitory shocks.

Table 1: Maximum Likelihood Estimates from the BN/UC Decomposition

	GDP	IP	Consumption	Personal Income
<u>Trend</u>				
Drift: μ	0.8326	0.2837	0.8667	0.8654
Standard deviation: σ_{η}	1.1697	1.8416	1.1481	1.0013
<u>Cycle</u>				
ϕ_1	1.3629	0.6445	---	-0.4432
ϕ_2	-0.7623	---	---	---
θ_1	---	---	0.6102	---
θ_2	---	---	0.6060	---
Standard deviation: σ_{ε}	0.6199	0.9153	0.9888	0.5789
Covariance: $\sigma_{\eta\varepsilon}$	-0.6884	-1.6815	-1.02734	-0.5504
Ratio: $\sigma_{\eta}/\sigma_{\varepsilon}$	1.8868	2.0122	1.1611	1.7499

Table 2: Estimated Trend and Cycle Variances from the BN/UC Decomposition

	GDP	IP	Consumption	Personal Income
σ_{τ}^2	16.9651	82.8565	15.5667	26.6780
σ_c^2	0.2774	1.4515	0.0828	0.0020

Table 3: Maximum Likelihood Estimates from the UC-0 Model

	GDP	IP	Consumption	Personal Income
<u>Trend</u>				
Drift: μ	0.8371	0.2876	0.8702	0.8648
Standard deviation: σ_{η}	0.6215	0.6859	0.0008	0.9271
<u>Cycle</u>				
ϕ_1	1.5110	1.6970	0.9639	0.7771
ϕ_2	-0.5858	-0.7172	---	---
θ_1	---	---	0.0138	---
θ_2	---	---	2.8255	---
Standard deviation: σ_{ε}	0.6400	0.5276	0.2867	0.4683
Ratio: $\sigma_{\eta}/\sigma_{\varepsilon}$	0.9711	1.3	0.0028	0.9797

Table 4: Estimated Trend and Cycle Variances from Alternative Decompositions

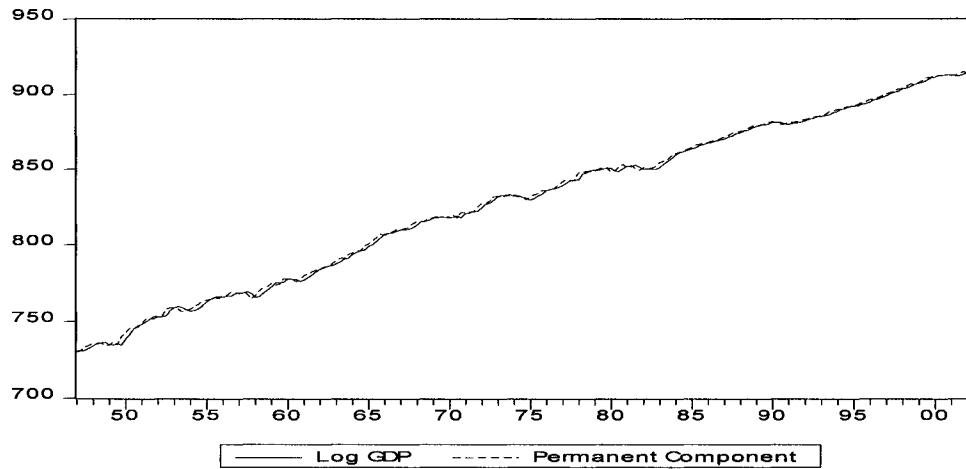
	GDP		IP	
	σ_{τ}^2	σ_c^2	σ_{τ}^2	σ_c^2
UC	16.9651	0.2774	82.8565	1.4515
UC-0	10.2425	3.3979	58.1012	12.8970
HP	11.9858	2.8344	80.4511	8.1136
BK	11.7482	2.6333	65.8562	11.4973

Table 4: Continued

	Consumption		Personal Income	
	σ_{τ}^2	σ_c^2	σ_{τ}^2	σ_c^2
UC	15.5667	0.0828	26.6780	0.0020
UC-0	1.05173	0.6670	25.9416	0.0381
HP	11.9367	1.8444	23.4160	1.6021
BK	11.5993	1.5988	24.3923	1.3605

Figure 1: Estimated Permanent and Transitory Components

Log GDP and the Permanent Component



The Transitory Component of Log GDP

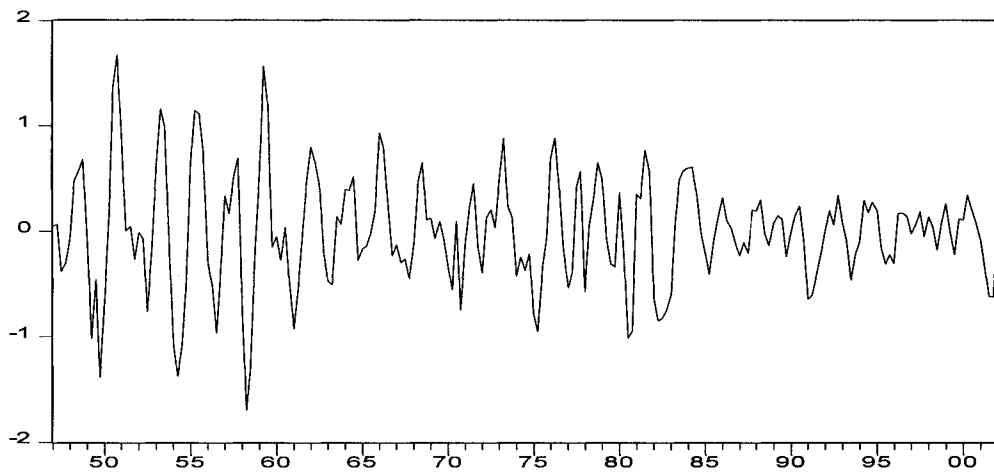
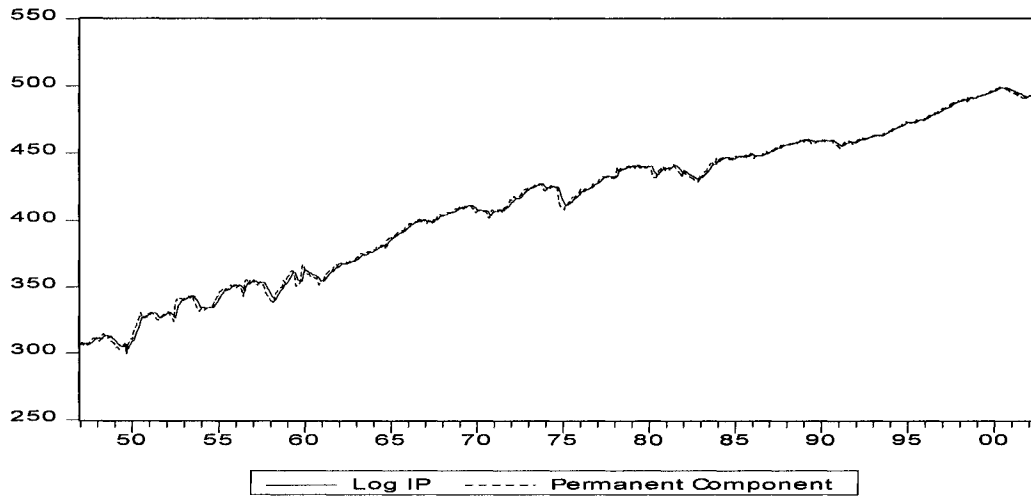


Figure 1: Estimated Permanent and Transitory Components (Continued)

Log IP and the Permanent Component



The Transitory Component of Log IP

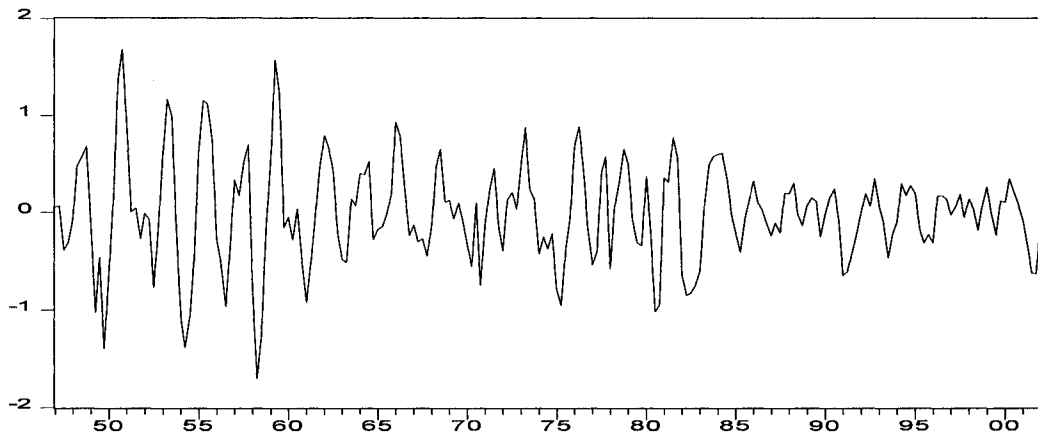
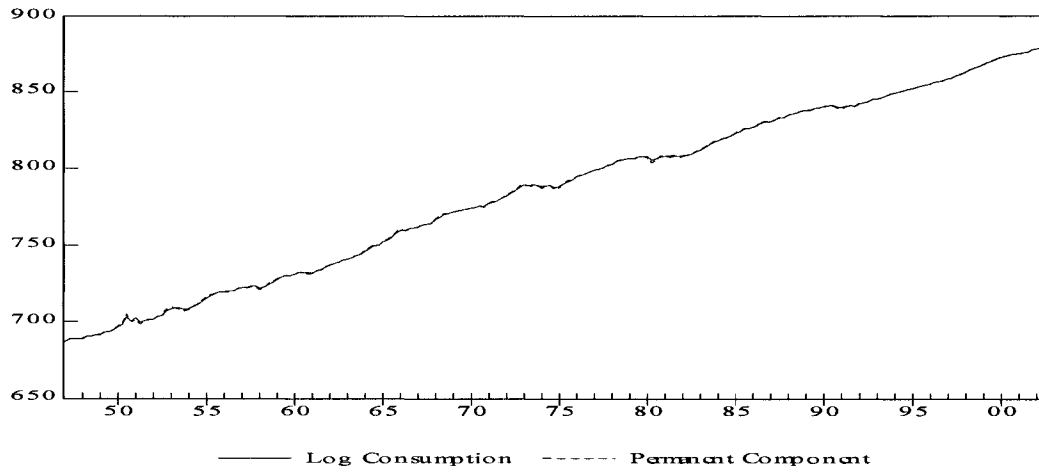


Figure 1: Estimated Permanent and Transitory Components (Continued)

Log Personal Consumption and the Permanent Component



The Transitory Component of Log Personal Consumption

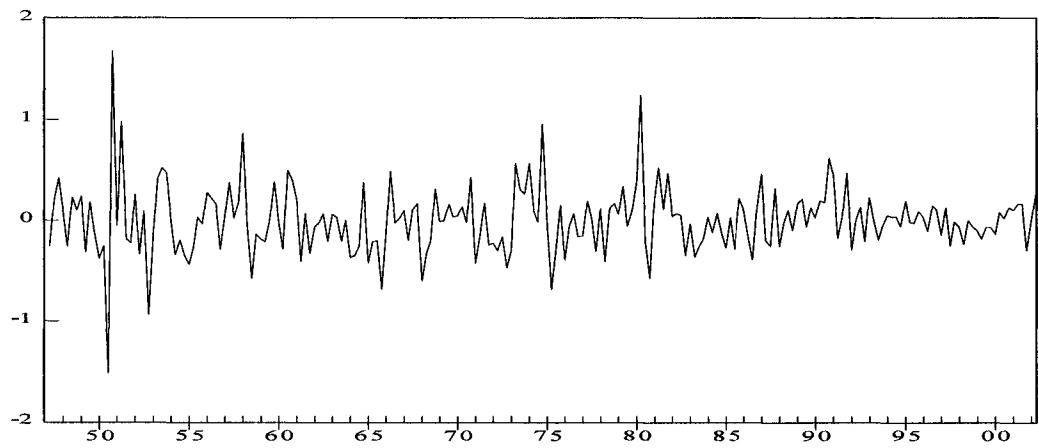
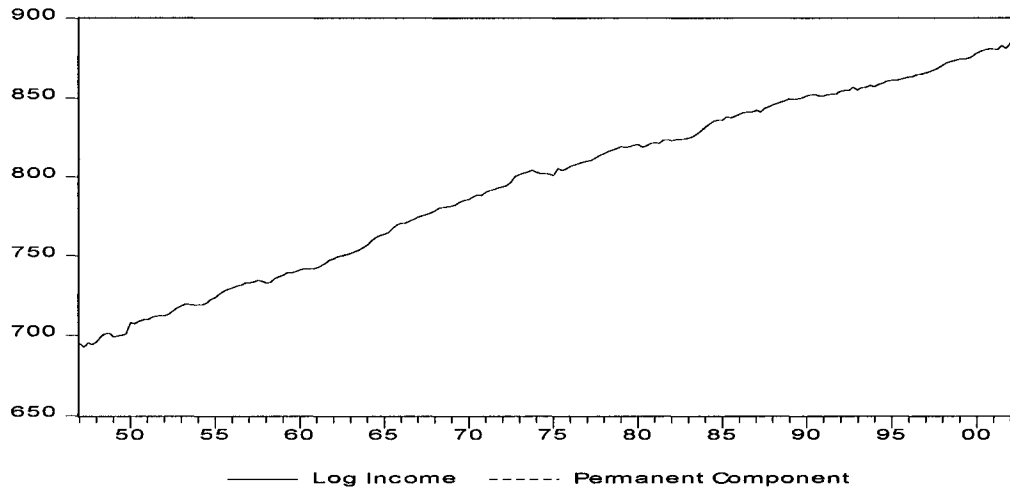


Figure 1: Estimated Permanent and Transitory Components (Continued)

Log Real Personal Disposable Income and the Permanent Component



The Transitory Component of Log Real Disposable Income

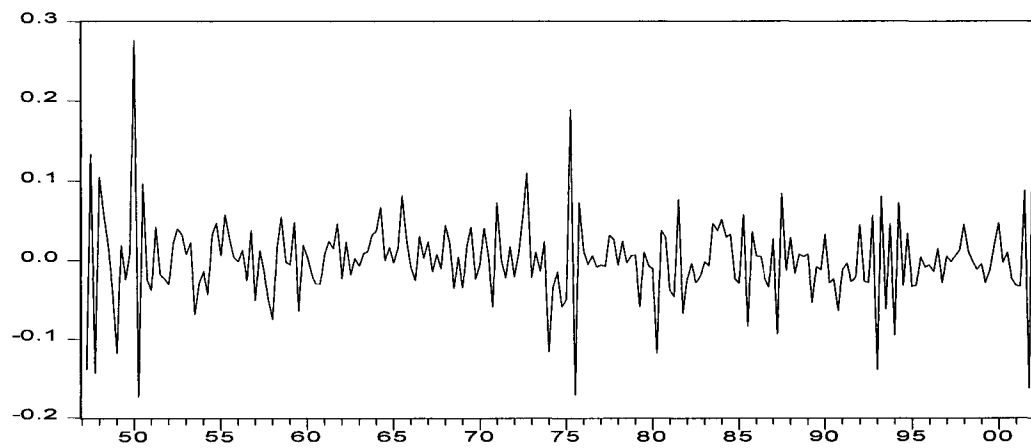
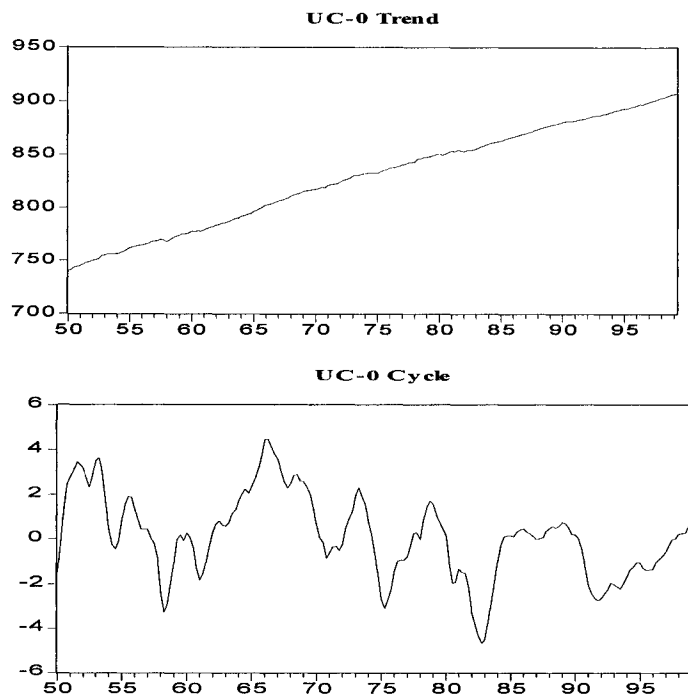


Figure 2: UC-0, Hodrick-Prescott and Baxter-King Decompositions of Log Real GDP

Permanent and Transitory Components from Restricted UC Model



Permanent and Transitory Components from the Hodrick-Prescott Filter

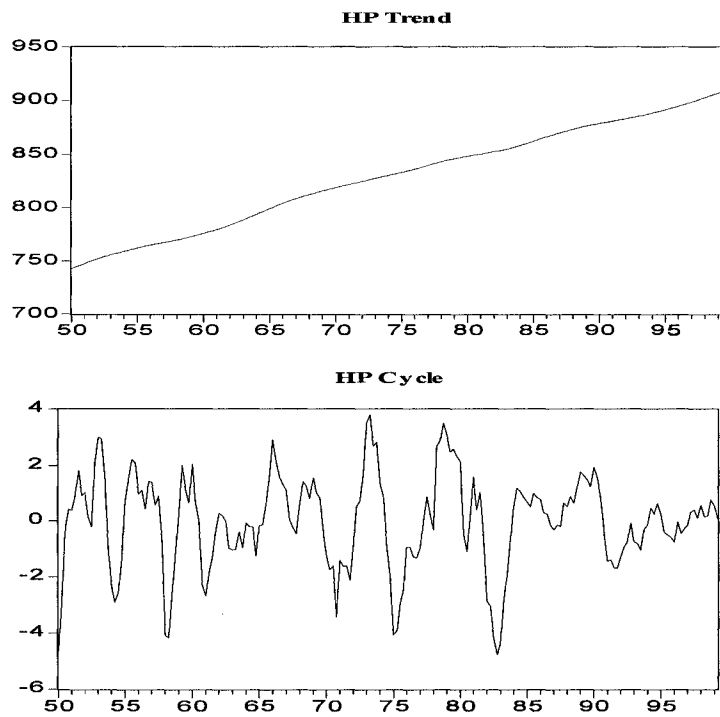
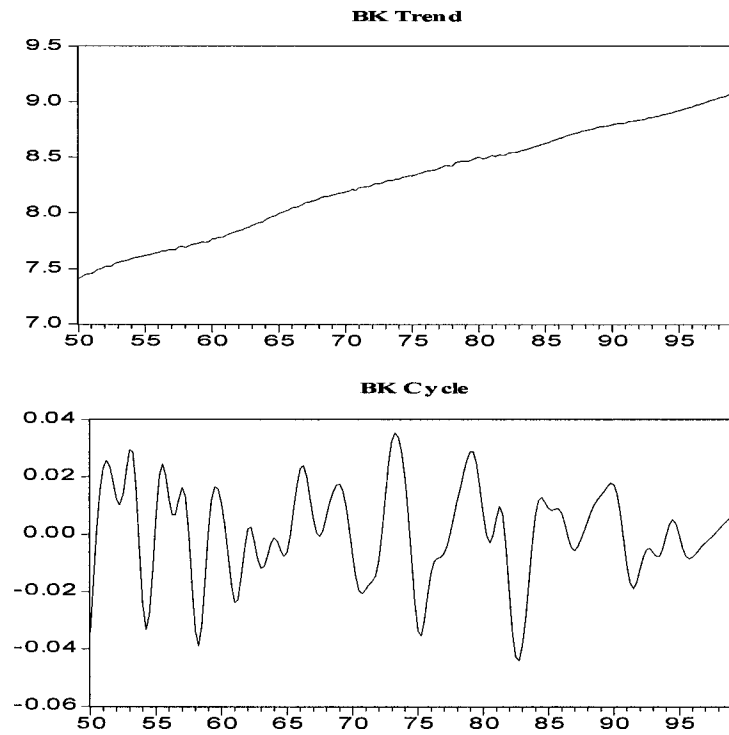


Figure 2: UC-0, Hodrick-Prescott and Baxter-King Decompositions of Log Real GDP (Continued)

Permanent and Transitory Components from the Baxter-King Filter



Study II: The Dynamics of Permanent and Transitory Components in International Business Cycles

1. Introduction

Business cycles, referred to as a “complex phenomenon”⁵ by Haberler (1968), are even more complex today than they were in 1936 when he finished the first draft of his book. Today, the globalization of financial markets and the decrease of trade barriers turns the attention from a single country’s upswings and downswings to the level of synchronization and linkages between nations’ business cycles. Yet it is not quite clear how the country’s business cycles are affected by the increase of international trade and financial integration. Whereas the logical outcome of increased interdependence among countries should be the synchronization of cycles, the theoretical and empirical evidence on synchronization is mixed.⁶ While we know some facts about the degree of interdependence between countries: for example that there is no evidence of increased correlation of outputs between developed countries, but the common international factor plays an important role in the business cycles of the developed countries – there is still a

⁵ Haberler (1968) writes: “Such a complex phenomenon as the business cycle, which embraces almost all parts of the economic system, does not easily lend itself to explanation by any one factor. Even if we assume from the beginning that the same explanation of the business cycle holds good in the highly industrialized countries of Western Europe and America as well as in industrially less developed countries such as New Zealand or Roumania, and in the twentieth century as well as at the beginning of the nineteenth – neither of which assumptions is by any means self-evident – it is not easy to speak of *the* cause of the business cycle.”

⁶ In the encyclopedia on business cycles and depressions, Dore (1997) presents the synchronization of international cycles as a stylized fact. At the same time, the studies of pairwise correlations among G7 growth rates by Doyle and Faust (2002a, 2002b) did not find any significant evidence of increased comovement between the countries’ outputs. However, the applications of dynamic factor models by Kose et. al (2003), and Gregory et. al (1997) find evidence of increased international business cycle linkages. Within the theoretical framework Krugman (1993) finds that trade integration can lead to a greater regional specialization and consequently to less output synchronization. Heathcote and Perri (2002) find that the trend towards financial globalization has been accompanied by a trend towards real regionalization. While for example, Imbs (2003) shows that countries’ cycles synchronize as countries grow richer and more diversified and Imbs (2004) that regions with stronger financial integration are more synchronized.

lot to learn. In this study I suggest a framework that is novel for the study of international business cycles and has proven to be fruitful in studying US economic fluctuations.

The purpose of this study is to measure international business cycles by means of extensive examination of the dynamics of trend and cycle that are common to the seven most developed nations of the world. The methodology presented in the study simultaneously measures two types of international comovement – comovement among countries' trend and comovement among countries' cycle. At the same time, the common trend and cycle are allowed to exhibit different behavior depending if they are in expansionary or recessionary phases of the international business cycle. The study also provides estimates on the importance of international trend and cycle in the business cycle fluctuations of each nation.

The growing empirical literature on international business cycles is comprised of a variety of mixtures of methodologies, country groupings, data sets, and data types. The methodologies vary from univariate models for international business cycles, where each of the country's time series are estimated separately and then the concluding remarks are drawn; to the estimation of the international comovement among the countries' time series through pairwise cross-correlation; to multivariate dynamic factor models or vector autoregressive models applied to three and up to sixty countries. The country-groups examined in the literature are mostly concentrated around G7 countries, OECD countries, or Euro zone countries. In the unique case as in Kose et al. (2003) sixty countries are used in the model which makes it the largest cross country data set used so far.

There are several shortcomings in the existing empirical research regarding the data length and frequency as well as the transformations applied to the series. Most of the research is conducted either for quarterly data that begin in the early 1970s or for annual data that is available from 1870. In the first case the sample period is not long enough to estimate important changes in the economy which could be interpreted as common shocks, for example an oil crisis of 1973 or a change to a floating exchange rate regime. In the second case, when annual data sets are used it allows researchers to extend their

study to a longer periods of business cycles, or to the inclusion of more countries.⁷ On the other hand, contractions can last as short as six months, and the annual data will not be able to capture these.

Another possible shortcoming of some empirical results is the use of a prior transformation of the series. Transforming the series is necessary because the variables that are commonly used to measure international business cycles, for example industrial production or real GDP, can possibly contain a nonstationary component. Thus to detrend the series they are either first differenced and the study is conducted for growth rates, or the cyclical component is isolated with the use of Hodrick-Prescott (1997) filter or Baxter-King (1999) filter. The interpretation of Hodrick-Prescott or Baxter-King filters depends on definition of the cyclical component. The use of those filters for nonstationary series has been criticized by Harvey and Jaeger (1993), Cogley and Nason (1995), and Murray (2003). It has been shown in the literature that detrending nonstationary series using the Hodrick-Prescott and Baxter-King filters can induce spurious cyclical behavior. Recognizing the existing controversy on the subject of those filters and to avoid the “minefield”⁸ it is common practice to simply first difference the series.

The study overcomes some of the existing shortcomings in the literature by adopting the following definition of the cyclical component: the cycle is the stationary component of the series and the shocks to the cycle are transitory. The trend or the permanent component is stochastic, nonstationary component of the series.

The ability of my model to differentiate between two different types of common shocks – permanent and transitory, is important for several reasons. First, from a theoretical point of view, the results of the study can help to understand what type of

⁷ Kose et. al (2003) examine annual real GDP series for 60 countries. Bordo and Helbling (2002) examine annual real GDP and industrial production historic series that begin in 1870. Basu and Taylor (1999) examine annual time series on real GDP, prices, real wages, exchange rate, consumption, investment, and the current account from 1870 to present.

⁸ Helbling and Bayoumi (2003) refer to the subject of detrending as “minefield” because of the existing controversy on the method of detrending applied. In this regard the authors examine first-differenced series in addition to Baxter-King detrended series. This is a common practice for the researchers. Gregory et. al investigate both first-differenced and Hodrick-Prescott filtered time series. Stock and Watson (2003) examine both first-differenced and Baxter-King filtered series.

shocks should be considered for international business cycle models,⁹ if the nature of the shocks should be permanent, transitory or some combination of both. Second, from an empirical perspective, we can learn more about the comovement and synchronization from the study of a panel data. Since the scope of the study is to estimate the effect of global shocks, whether those affect the permanent or transitory component of business cycles and how each of the countries is affected by the shocks – the results are important for calculation of stylized facts, optimal policy recommendations, and business cycle forecast improvement.¹⁰

The study presents a multivariate unobserved components model with Markov regime switching applied to quarterly real GDP of G7¹¹ countries. The time series examined cover the period from 1960 first quarter to 2002 last quarter. The methodology applied measures two major characteristics of business cycle fluctuations – the comovement among macroeconomic indicators and the asymmetry between the expansionary and recessionary phases.

The comovement in the permanent and transitory components of output is measured by two distinct unobserved components, which correspond respectively to an unobserved component that is common to the individual permanent components of the countries' output and an unobserved component that is common to the individual transitory components of the series. The first common component is defined as an international trend and the second common component is defined as an international

⁹ Basu and Taylor (1999) state that theories on business cycles make two sorts of claims: 1) what types of shocks are more important in disturbing the economy, and 2) what is the economic structure necessary for propagating these shocks. In international business cycles framework an example of first, is the study by Ahmed et. al (1993) who model world wide productivity shock as well as several country specific shocks to estimate the importance of those shocks within an open economy real business cycle model. An example of the second is Backus, Kehoe and Kydland (1992) work which examines the ability of open economy real business cycle model to account for both international and intranational comovements.

¹⁰ Doyle and Faust (2002a) note that national economic policies should take into account forecasts for conditions abroad in formulating forecasts for their domestic economies. Filardo and Gordon (1994) note that the potential policy uses of the multivariate time series study capturing comovement are: designing optimal policies, as currency unions; improvement of forecasts of one nation's business cycle when other national business cycle phase data is available; calculating stylized facts which are useful for macroeconomists who are trying better understand the economy by matching models to the data.

¹¹ Real GDP is considered a sufficient variable to measure business cycle fluctuations. The heavy majority of the papers base the international business cycle estimation on the countries' real GDP series. Some also investigate industrial production index. In addition to the real output Gregory et al. (1997), as well as Kose et. al (2003) include consumption and investment of the countries in to the models. The G7 grouping of the countries is chosen as a cluster most representative developed country.

cycle. The asymmetric behavior of the international trend is modeled with Hamilton (1989) two-state Markov regime switching, typically characterized as switching between expansionary and recessionary phases of business cycles. In this model the trend is allowed to have two different growth rates depending on the state of the economy. The asymmetry of the international cycle is modeled with two-state Markov regime switching according to Friedman's (1964) *plucking*¹² model. His study suggests that output is "bumping along the ceiling of maximum feasible output except that every now and then it is plucked down by a cyclical contraction," thus contractions have transitory effect on the economy. Following Kim and Nelson's (1999) methodology applied to estimate Friedman's plucking model for US real output, the cyclical component is modeled to have mean zero during the expansion and to be plucked down during the contraction.

The simultaneous analysis of comovement and asymmetry for the US economic indicators conducted by Kim and Murray (2002) suggest that contractions in US are influenced by both permanent and transitory components, however are mostly driven by the transitory component. Their study builds on the previous work by Wynne and Balke (1992), Beaudry and Koop (1993), and Sichel (1994) that show that in addition to traditional expansion and contraction phases, US business cycles contain a third phase characterized as high-growth recovery. The recovery phase follows the contraction phase and precedes the expansion phase. In this regard, Kim and Murray's (2002) comprehensive study of the US business cycle dynamics concludes that US business cycles consist of three phases: recession, partial recovery, and normal growth.

The contribution of this study is that it is the first study to simultaneously analyze the comovement and asymmetry for international time series differentiating between international permanent and transitory components. I find that the dynamics of the international business cycle are different from the dynamics of single country business cycles. Comparing international business cycles to US business cycles I conclude that

¹² Friedman describes the 'plucking' behavior of business cycles as follows: "Consider an elastic string stretched taut between two points on the underside of a rigid horizontal board and glued lightly to the board. Let the string be plucked at a number of points chosen more or less at random with a force that varies at random, and then held down at the lowest point reached. The result will be to produce a succession of apparent cycle in the sting whose amplitudes depend on the force used in plucking the string."

the international business cycles of G7 countries lack the recession and recovery phases, since I do not find any evidence of an international transitory component. Moreover, the growth rate of the international permanent component is positive in both states and corresponds to 2.88% and 1.32% respectively. Thus, I conclude that the international business cycle of G7 countries has two phases, a high-growth phase and a low-growth phase.

I find that Japan is the most sensitive to international permanent shocks, followed by France, Canada, Italy, US and UK. Germany is the least sensitive to international permanent shocks.

I also find that during 1960-2002 there has been only one shock that was able to influence all seven developed countries. The shock to the international permanent component caused the international permanent component to switch from a high-growth state to a low-growth state. This shock happened in 1973 and is traditionally associated with the first oil crisis. There have been many other explanations for the productivity slowdown that occurred in the early 1970s. While the study is not taking sides on what the major cause of the slowdown was, it estimates that the slowdown occurred due to in the second quarter of 1973.

The outcome of the study raises a question: “Are there international business cycles at all?” If there are international business cycles, then they are characterized as common to the country’s trend not their common cyclical fluctuations. If we accept Doyle and Faust’s (2002a) argument that the causes for increase in synchronization are “an increase in the amount of common variation in the economies” or “a decrease in idiosyncratic variation,” then there has been no increase in synchronization among G7 countries. Since there is a single shock of 1973 that affected the international business cycles of G7 countries and the recessionary periods are purely country specific, the idiosyncratic volatility did not necessarily decrease.

The existing empirical research on international business cycles notably lacks a unified framework for analyzing the dynamics of international business cycles. An unobserved components model that measures the comovement and asymmetry of common permanent and transitory components is advocated as a general framework for

measuring international business cycles. The model is clear in its definition of trend and cycle and does not require prior detrending of the series.

The rest of the study is organized as follows. The next section compares the methodologies that have been applied to estimate comovement and asymmetry of the US business cycles to the similar application in measuring international business cycles pointing out what can be still added to the already existing research. Section 3 presents the data and the unobserved components model, which is proposed as a general framework for measuring international business cycles. Section 4 discusses the empirical results from the estimated models. Section 5 concludes the study and suggests possible extensions.

2. Business Cycle Definitions and Methodologies

The purpose of this section is to compare the methodologies developed to measure the US business cycle with similar applications in measuring international business cycles, to point out the existing gaps in international business cycles applications, and why it is important to fill them.

Two commonly accepted empirical characteristics of the US business cycles are the comovement among the macroeconomic variables and the existing asymmetry between the expansionary and recessionary phases of the business cycles. Mitchell (1927) states that “Business contraction seems to be a briefer and more violent process than business expansion.” Burns and Mitchell (1946) define business cycle as follows:

“...a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.”

Though this definition was written based on an empirical investigation of the US macroeconomic activities, it can be expanded to an international framework. The studies on international business cycles are focused on the issue of synchronization of business

cycles across nations, which translates into comovement of major economic indicators of those nations. At the same time, in the case when each country's expansionary periods differ from the recessionary periods, then if there is a common cycle there is a high probability that the common cycle also will exhibit that asymmetry.

Earlier in the study the cycle was defined as the stationary deviations from the stochastic trend. In this regard, an unobserved components model is the accepted choice to decompose the series into trend and cycle. The foundation of the econometric estimation of permanent and transitory components is in papers by Beveridge and Nelson (1981) and Nelson and Plosser (1982). Beveridge and Nelson (1981) demonstrate that any nonstationary time series can be represented as a sum of two components – a stationary process with zero mean, and the random walk with the same rate of drift as the original series. Nelson and Plosser (1982) show that most macroeconomic time series follow nonstationary process and apply the Beveridge-Nelson decomposition to measure the importance of the permanent component. They find that most of the variation in output is due to the variance of the permanent component.

The unobserved components model was first implemented by Harvey (1985) and Clark (1987) to measure the relative importance of permanent and transitory shocks in US business cycles. The model applied was a univariate linear model that was unable to measure comovement or asymmetry emphasized by Mitchell (1927) and Burns and Mitchell (1946). However, as it is discussed later in the section, the unobserved components model can be extended to account for both comovement and asymmetry, and still be extremely valuable for its trend-cycle decomposition. The unobserved components model has been shown to be a fruitful framework for investigating US business cycle fluctuations, but it has not been well utilized in measuring international business cycles.

2.1 Comovement Among Macroeconomic Variables

Comovement of economic time series has played an important role in analyzing and forecasting business cycle fluctuations. Burns and Mitchell (1946) define business cycle as expansions and contractions “occurring at about the same time in many

economic activities.” Composite Indexes of Coincident and Leading Economic Indicators, initially developed by Mitchell and Burns (1938), are used up to this day by the Department of Commerce (DOC) to summarize the current state of macroeconomic activity.

Stock and Watson (1989, 1991, and 1993) suggest an explicit probability model as an alternative framework to capture the comovement among the coincident series that are used by DOC to compute the index of Coincident Economic Indicators (CEI). They apply a linear dynamic factor model, where the comovement among the variables is expressed through a single unobserved component common to all the series. The common unobserved component estimated by Stock and Watson (1989, 1991, and 1993) closely coincided with the Index of CEI computed by the DOC.

The notion of comovement between different nations’ business cycles started as early as Mitchell’s (1927) work, whose empirical investigation shows that a large group of nations’ business cycles roughly coincide. The international comovement was examined in univariate, bivariate¹³ and multivariate models. Multivariate models represent a more general framework for investigating comovement among different countries.

The dynamic factor model implemented by Stock and Watson (1989, 1991, and 1993) to analyze comovement across economic variables was reformulated as multi-country dynamic factor model to measure the international comovement among the aggregate variables of several countries. It was estimated by Gregory et al. (1997) to measure world-specific and country-specific components and by Kose et al. (2003) to measure world, regional, and country specific components.¹⁴ Both of these analyses use the cyclical components of the series, which are isolated by Hodrick-Prescott filter and first-differencing in Gregory et al. (1997) and by first-differencing in Kose et al. (2003).

¹³ Within the univariate framework Artis et al. (1997) apply Bry and Boschan (1971) National Bureau of Economic Research procedure to estimate reference dates of business cycles. Examples of bivariate analysis are: Doyle and Faust (2002a, 2002b) examine the comovement among the growth rates of G-7 countries estimating the pair wise correlation fluctuations; Gerlach (1988) measures correlations of two series at different frequencies for industrial production indices of OECD countries.

¹⁴ Gregory et al (1997) apply dynamic factor model and Kalman filtering techniques. Kose et al. (2003) employ a Bayesian dynamic latent factor model.

An alternative methodology to measure comovement is an application of VAR analysis, for example Stock and Watson (2003) develop a Factor Specific Vector Autoregression (FSVAR) to estimate international comovement as well as the origins of the changes in international comovement. An interesting approach is taken by Carvalho and Harvey (2003) and Luginbuhl and Koopman (2004), who apply an unobserved components decomposition to measure the convergence in countries' permanent and transitory components. The difference between Carvalho and Harvey (2003) and Luginbuhl and Koopman (2004), and the studies conducted by Gregory et al. (1997), Kose et al. (2003), Stock and Watson (2003) is that the last group examines already detrended variables and focuses only on the dynamics of the international cyclical component.

2.2 Asymmetry in Trend

The asymmetric nature of business cycles was first highlighted by Mitchell (1927) who emphasized that the business cycles contractions are fundamentally different from expansions. Using a Markov process framework, Neftci (1984) presents the first statistical tests on asymmetric behavior of time series, which triggered the development of nonlinear univariate models to incorporate asymmetry. Since then, several different approaches have been developed to model business cycle asymmetry.

Asymmetry in the permanent component was modeled by Hamilton (1989). In his model the trend is governed by an unobserved Markov switching state variable and the economy faces one of two states: positive growth, expansion, or negative growth, contraction. Hamilton (1989) finds that the business cycle is characterized by a recurrent pattern of shifts between recessionary state and a growth state.

The simultaneous examination of comovement among time series and the asymmetry in the phases of business cycle resulted in more accurate estimation of business cycles and a coincident index that is remarkably similar to that reported by the National Bureau of Economic Research (NBER). The advantage of this synthesis was pointed out by Diebold and Rudebusch (1996) and estimated by M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998).

Within an international business cycles framework Krolzig (2003) applies Hamilton's (1989) Markov switching model in a VAR framework to construct turning point chronologies for the European business cycle. Filardo and Gordon (1994) develop a multivariate model with two-state Markov regime-switching to capture phase comovement in international business cycles.

2.3 Asymmetry of the Cyclical Component

Asymmetry in the cyclical component was first proposed by Friedman (1964). Friedman (1964) claims that each contraction has the same amplitude as the next expansion and there is no connection between the amplitude of the expansion and the amplitude of the next contraction. His study suggests that output is constrained by a ceiling of maximum feasible output and is occasionally plucked down by a cyclical contraction. Friedman's *plucking* model implies that contractions are asymmetric and that their effect on the economy is transitory.

Kim and Nelson (1999a) present an unobserved components model that decomposes real output into permanent and transitory components and accounts for Friedman's (1964) plucking type of asymmetry in the transitory component. Kim and Nelson's (1999) model reflects the "peak reversion" or asymmetry in the persistence of shocks highlighted by Beaudry and Koop (1993) and Sichel (1994). They conclude that the US quarterly real GDP indeed has a ceiling level which is determined by the stochastic trend component and the transitory component is plucked down during recessionary periods, after which output quickly recovers to its trend level.

Similar to Hamilton's (1989) model, Kim and Nelson's (1999) model is univariate, therefore does not take into account the comovement among aggregate variables. In this regard, Kim and Murray (2002) propose a generalized regime-switching dynamic factor model, which allows for both comovement and asymmetry in permanent and transitory components. They examine four coincident series, formerly examined by Stock and Watson (1989, 1991, and 1993), and find that each of the US recessions differ in terms of the contribution of the common permanent and transitory components.

From the perspective of international business cycles, Kim and Nelson's (1999) *plucking* model was applied by Mills and Wang (2002) to estimate asymmetry in the transitory component of G7 real GDP series. Mills and Wang (2002) find evidence for downward plucks, as each country's output is influenced by negative asymmetric shocks, but the timing and/or depth of the downward plucks of countries did not coincide. Mills and Wang's (2002) results are purely univariate. Kim and Nelson (2001), who test for Markov regime switching in univariate and multivariate unobserved components models, conclude that the evidence of asymmetry in business cycle is much more compelling in the multivariate tests. Nevertheless, the asymmetry of the transitory component of international business cycles has not been examined within a multivariate framework.

3. Unobserved Components Model

The model presented in this section is a nonlinear multivariate unobserved components model that can serve as a generalized multivariate framework to measure the significance of common transitory and permanent components in international business cycle fluctuations. This framework is suitable and novel. It comprises two important features in measuring international business cycles: a) the decomposition of integrated series into stochastic trend and cyclical components emphasized in Carvalho and Harvey (2003), Luginbuhl and Koopman (2003), and b) dynamic factor model which is important for the isolation of internationally common components from country specific components, and series specific, idiosyncratic components, which was the primary focus in Gregory et al. (1997) and Kose et al. (2003). To account for the asymmetric behavior of international trend and cycle, the unobserved components model presented below also incorporates regime switching between the expansionary and recessionary phases of the world economy for the common permanent and transitory components. The simultaneous examination of asymmetry in the permanent and transitory component of

the series together with the comovement among them was formerly examined for a single country case¹⁵ but has not been applied to international fluctuations.

Unobserved components model is characterized by the following equations:

$$Y_{it} = \gamma_i T_t^w + \alpha_i c_t^w + \tau_{it} + c_{it} \quad (1)$$

$$\phi^\tau(L)\Delta T_t^w = \mu^w + v_t, v_t \sim iidN(0,1) \quad (2)$$

$$\phi(L)c_t^w = u_t, u_t \sim iidN(0,1) \quad (3)$$

$$\tau_{it} = \mu_i + \tau_{it-1} + \omega_{it}, \omega_{it} \sim iidN(0, \sigma_{\omega_i}^2) \quad (4)$$

$$\psi_i(L)c_{it} = \varepsilon_{it}, \varepsilon_{it} \sim iidN(0, \sigma_{\varepsilon_i}^2) \quad (5)$$

where Y_{it} is 100 times log of individual time series, $i = 1, \dots, N$, and N is number of time series. In this model number of series is equal to number of countries. The model can be easily extended to include more series. Y_{it} is decomposed into T_t^w , the common stochastic trend, τ_{it} , the series specific, idiosyncratic stochastic trend, c_t^w , the common cyclical component, and c_{it} , the series specific, idiosyncratic cyclical component. Throughout the study, T_t^w and c_t^w will be referred to as international permanent component and international transitory component or as international trend and cycle. Both international cycle and idiosyncratic cycle are assumed to follow an autoregressive process. γ_i and α_i are factor loadings for the international trend and international cycle respectively. γ_i indicates the extent to which each series are affected by common permanent component, while α_i indicates the extent to which each series are affected by common transitory component. For identification of the model the variances of common components are normalized to one.

The model is estimated in differences and is written in deviations from means.

$$\Delta y_{it} = \gamma_i \Delta \tau_t^w + \alpha_i \Delta c_t^w + z_{it} \quad (6)$$

¹⁵ Kim and Murray (2002) measure the asymmetric behavior of common permanent and transitory components of four US coincident indicators.

where $\Delta y_{it}, \Delta \tau_t^w, z_{it}$ are defined as $\Delta y_{it} = \Delta Y_{it} - \Delta \bar{Y}_i$, $\Delta \tau_t^w = \Delta T_t^w - \delta$, $z_{it} = \Delta c_{it} + \Delta \tau_{it}$. I allow $\Delta \tau_t^w, \Delta c_t^w$ and z_{it} to follow the processes described in equations (7), (8) and (9) respectively. The international permanent component is subject to Hamilton (1989) regime switching.

$$\phi^\tau(L)\Delta \tau_t^w = \mu_{S_{1t}} + \nu_t, \quad \nu_t \sim N(0,1) \quad (7)$$

$$\mu_{S_{1t}} = \mu_0(1 - S_{1t}) + \mu_1 S_{1t}$$

$\mu_{S_{1t}}$ is defined as $\mu_{S_{1t}} = \mu^w - \delta$. The international transitory component is subject to Kim and Nelson (1999a) regime switching.

$$\phi^c(L)c_t^w = \lambda_{S_{2t}} + u_t, \quad u_t \sim iidN(0, \sigma_{u, S_{2t}}^2) \quad (8)$$

$$\sigma_{u, S_{2t}}^2 = \sigma_{u,0}^2(1 - S_{2t}) + \sigma_{u,1}^2 S_{2t}$$

$$\lambda_{S_{2t}} = \lambda S_{2t}$$

The idiosyncratic components z_{it} follow autoregressive processes such as:

$$\psi_i(L)z_{it} = \eta_{it}, \quad \eta_{it} \sim iidN(0, \sigma_i^2) \quad (9)$$

S_{1t} and S_{2t} are Markov switching state variables that switch between 0 and 1 and have q_1, q_2 and p_1, p_2 transition probabilities such as:

$$S_{1t} = \{0,1\}, \quad \Pr[S_{1t} = 0 | S_{1,t-1} = 0] = q_1, \quad \Pr[S_{1t} = 1 | S_{1,t-1} = 1] = p_1$$

$$S_{2t} = \{0,1\}, \quad \Pr[S_{2t} = 0 | S_{2,t-1} = 0] = q_2, \quad \Pr[S_{2t} = 1 | S_{2,t-1} = 1] = p_2$$

$\mu_{S_{1t}}$ is the deviation of ΔT_t^w from its long-run growth δ . The world trend undergoes low-growth and high-growth periods when $\delta + \mu_0 > \delta$ ($S_{1t} = 0$) and $\delta + \mu_1 < \delta$ ($S_{1t} = 1$) respectively. $\lambda_{S_{2t}}$ is an asymmetric shock to world cycle, which is equal to zero during the expansionary periods when $S_{2t} = 0$. In this case the international economic fluctuations are near their potential output level or international trend. The common permanent component is a ‘‘ceiling level’’ for the common transitory component. $\lambda_{S_{2t}} = \lambda$ is expected to be negative during the recessionary periods when $S_{2t} = 1$ and the common cycle is hit with transitory shock. In Friedman’s (1964) terminology, λ is the size of the

pluck for the common transitory component, so that the transitory component is plucked down during recessionary periods.

The variance of the symmetric shock to the common transitory component is allowed to be different during recessions and expansions. Accordingly the disturbances u_t are heteroskedastic and follow a Markov-switching process. The variance of u_t is normalized to one when $S_{2t} = 0$, and equal to $\sigma_{u,S_{2t}}^2 = \sigma_{u,1}^2 \equiv \sigma_u^2$ when $S_{2t} = 1$.¹⁶ The variance of the common permanent component is normalized to one in both regimes. The model in which the disturbances to the common trend are heteroskedastic and follow the Markov-switching process was also estimated. However the estimated common trend was consistent with the common trend that contains homogenous disturbances with normalized variance.

To estimate the parameters as well as the unobserved components of the model, I employ Kim's (1994) approximate maximum likelihood estimation algorithm. The exact calculation of the Gaussian likelihood function is not possible because the state variables S_{1t} and S_{2t} are unobserved. The state-space representation and the estimation of the model are presented in the Appendix. The last section of the Appendix demonstrates how I calculate δ and construct T_t^w from $\Delta\tau_t^w$.

4. Empirical Results

4.1 Data

The model presented in section 3 is estimated for the log of quarterly real GDP for the G7 countries: Canada, France, Germany, Italy, Japan, UK and US. The time series cover the period 1960:1 to 2002:4, are seasonally adjusted and the base year is 1995. Using Dickey-Fuller GLS (DF-GLS) test I fail to reject the null of unit root for the real

¹⁶ Lastrapes (1989) and Lamoureux and Lastrapes (1990) show that the failure to allow for regime shifts lead to an overestimation of the persistence of the variance of a series. Under Markov-switching heteroskedasticity the unconditional variance is subject to abrupt shifts between different states. Hamilton and Susmel (1994) suggest that the long run variance dynamic by regime-shift thus Markov-switching heteroskedasticity is appropriate for modeling low frequency data over a long period of time.

GDP series at 10% significance level thus the series are integrated of first order.¹⁷ The nominal GDP series and the implicit price indices of Canada, France, Germany, Japan, UK and US are taken from OECD Quarterly National Accounts. Italian nominal GDP and the consumer price index are from IFM/IFS database. The GDP series of Canada and France were available only from 1961 and 1978 respectively. The missing observations at the beginning of the examined period were constructed using GDP volumes of those countries following Stock and Watson (2003). The GDP volume series are from Datastream database. The GDP series for Italy are deflated with consumer price index because of the inconsistency in the existing implicit price deflator series for Italy.¹⁸

4.2 Common Transitory Component

The maximum likelihood estimates for the regime switching dynamic factor model (1)-(5) are presented in Table 1. Figures 1 and 2 present the common transitory component and the smoothed probability of contraction for the common transitory component. At first glance, there are abrupt contractions in the transitory component in 1963, 1968 and 1975. However the transition probability p_2 and the downward pluck, λ are statistically insignificant. The factor loadings of the common transitory component, α_i , are also insignificant with exception of France and UK.¹⁹ Thus there is no evidence of asymmetry in common transitory component or the significance of the common transitory component itself for G7 countries.

The essence of Friedman's *plucking* model is that output cannot exceed a ceiling level, but it is plucked down during recessions, suggesting that recessions have a purely

¹⁷ Elliot et al. (1996) DF-GLS unit root test with Ng and Perron (2001) Modified Akaike Information Criterion (MAIC) lag selection is conducted for each of the series. DF-GLS is a state of the art unit root test that together with MAIC has good size and power. It fails to reject the null of unit root for the series at 10% significance level.

¹⁸ The GDP deflator for Italy indicates 1.3% deflation from 1969 fourth quarter to 1970 first quarter. There is no evidence to be found for a deflationary period in Italy in late 1960s or early 1970s. Early 1970s was an inflationary period for most of the developed countries.

¹⁹ The joint hypothesis that the transitory factor loadings, α_i are zero can not be tested because under the null hypothesis that $\alpha_i = 0$ for all i , the parameters associated with common transitory component, c_i^* , in equation (6) are not identified. Thus the distribution of the test statistic in presence of the nuisance parameters that exist only under the alternative hypothesis is unknown for the state-space model. For further discussion of this problem refer to Hansen (1996) and Garcia (1998). However standard distribution theory can be applied to test the individual hypothesis that $\alpha_i = 0$ for each $i = 1, \dots, 7$.

transitory effect on the economy. Kim and Nelson (1999a) as well as Kim and Murray (2002) find that the US cyclical component is well characterized by plucking and the plucks of the transitory component coincide with NBER recessionary dates.²⁰ Mills and Wang (2002) estimate Kim and Nelson's (1999) plucking model for G7 real GDP series. They find that each G7 country's transitory component exhibits asymmetric behavior and is plucked down, however those periods do not coincide. According to the OECD (2002) chronology of business cycles, as well as business cycles chronology dates reported by Economic Cycle Research Institute (ECRI),²¹ none of the recessionary dates for the seven countries during 1960-2002 coincide. Mills and Wang's (2002) results and the recession chronology dates for G7 countries support the fact that those countries do not share any common recessions and suggests that the countries do not have a common transitory component, which is in accordance with my finding. The lack of evidence of a common transitory component among G7 countries leads to a different specification of the model that suppresses the common transitory component and examines common permanent component only. This new specification of the model is discussed in the next section.

4.3 Common Permanent Component

Due to the lack of evidence of an international transitory component, I restrict the model in (1)-(5) so that the factor loadings of common transitory component equal to zero. In this case equations (1) to (5) collapse to (1'), (2'), (3') and (4').²² Model, which will be discussed primarily, consists of a common permanent component only, which is subject to asymmetric shocks.

$$Y_{it} = \gamma_i T_t^w + \tau_{it} + c_{it} \quad (1')$$

$$\phi^r(L)\Delta T_t^w = \mu^w + v_t, \quad v_t \sim iidN(0,1) \quad (2')$$

$$\tau_{it} = \mu_i + \tau_{it-1} + \omega_{it}, \quad \omega_{it} \sim iidN(0, \sigma_{\omega_i}^2) \quad (3')$$

²⁰ Kim and Murray (2002) find that five of the six recessions that occurred in the period from 1959-1998 contain a transitory component with timing that coincides with NBER recessionary dates.

²¹ ECRI reports NBER business cycles chronology dates for the US and uses NBER methodology to date the business cycles of the other countries.

²² The restricted model (1')-(4') can not be tested against the unrestricted model (1)-(5) for the reason explained in footnote 16 of the previous subsection.

$$\psi_i(L)c_{it} = \varepsilon_{it}, \varepsilon_{it} \sim iidN(0, \sigma_{\varepsilon_i}^2) \quad (4')$$

In differences deviations from means the estimated model is:

$$\Delta y_{it} = \gamma_i \Delta \tau_t^w + z_{it} \quad (5')$$

where $\Delta \tau_t^w$ and z_{it} are defined as in (6) and have the same structure as it is presented in (7) and (9).

$$\phi^r(L)\Delta \tau_t^w = \mu_{S_t} + \nu_t, \nu_t \sim N(0,1) \quad (6')$$

$$\mu_{S_t} = \mu_0(1 - S_t) + \mu_1 S_t$$

$$\psi_i(L)z_{it} = \eta_{it}, \eta_{it} \sim iidN(0, \sigma_i^2)$$

The maximum likelihood estimates for the model (1')-(4') are presented in Table 2. The state variable is defined as $S_t = \{0,1\}$ where $q = \Pr[S_t = 0|S_{t-1} = 0] = 0.9854$ and $p = \Pr[S_t = 1|S_{t-1} = 1] = 0.9970$. The transition probabilities of both regimes are close to unity, therefore both states are very persistent and the switch from one state to another is very unlikely. μ_0 is positive and corresponds to the $\Pr[S_t = 0|S_{t-1} = 0]$ phase of international business cycles. μ_1 is negative and corresponds to the $\Pr[S_t = 1|S_{t-1} = 1]$ phase of international business cycles. The growth rate of the international trend during state zero²³ is $\delta + \mu_0 = 2.88\%$. The growth rate of the international trend during state one is $\delta + \mu_1 = 1.32\%$. It should be noted that the growth rate of the international trend in both states is positive. Since the trend would be expected to have a negative growth rate during the recessionary period, the two phases of the international trend component can not be interpreted as expansion and recession. The international trend component switches from a high-growth rate of 2.88% to a low-growth rate of 1.32%, which means that the two phases of the international trend should be interpreted as having high-growth and low-growth phases. Thus, the international trend will undergo a change in slope when switching from one phase to the other but the slope will always remain positive.

²³ For the simplification of language, state zero and state one are referred to as $\Pr[S_t = 0|S_{t-1} = 0]$ and $\Pr[S_t = 1|S_{t-1} = 1]$ respectively.

The international trend and the smoothed probability of state one for the international trend are shown in Figures 3 and 4.²⁴ Figure 4 illustrates that the model estimated only one switch when the international trend changes from a high-growth regime to a low-growth regime. The switch occurs when $p > 0.5$, which corresponds to the second quarter of 1973. The switch of the international trend from a high-growth regime to a low-growth regime is reflected in Figure 3. As was already mentioned, the international trend has a change of slope from steep to moderate.

There are three major outcomes to reveal. First, the international business cycle characteristics are different from the characteristics of individual countries' business cycles. It consists of two phases, a high-growth phase and a low-growth phase, which can not be interpreted as traditional expansion and contraction phases. Second, the model estimated the productivity slowdown that occurred in 1973 as a switch from a high-growth phase to a low-growth phase in the international trend. Third, among the seven developed countries, Japan is the most sensitive to international shocks and Germany is the least sensitive country. The significance of these results is discussed below.

Result 1:

Hamilton's (1989) univariate study as well as the multivariate studies by Diebold and Rudebusch (1996), M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998a), follow the traditional view of a two-phase business cycle. They find that US business cycles are characterized by two phases: a positive growth phase and a negative growth phase that affect the permanent component and correspond to expansion and contraction phases respectively. However more thorough empirical studies on US business cycles suggest that the US business cycle is well characterized by three phases.

The idea that the business cycles have three phases was first discussed in Burns and Mitchell (1946). In addition to expansion and contraction they refer to a "revival" phase which "merges into the expansion phase of the next cycle." The univariate studies of US fluctuations by Wynne and Balke (1992), Beaudry and Koop (1993) and Sichel

²⁴ There is no difference in the estimated common trend from models (1)-(5) and (1')-(4'). Restricting the model to not having an international cycle does not change the estimated international permanent component.

(1994) and the multivariate study by Kim and Murray (2002) find that there is indeed a “revival” phase. The concept behind the three-phase business cycle is that negative shocks cause only temporary declines in output from its normal growth. In this regard, Beaudry and Koop (1993) state that the effects of negative shocks are temporary and the effects of positive shocks are permanent.

Wynne and Balke (1992) find a statistically significant relationship between the growth rate of US output in the first twelve months after trough and the depth of the decline in output from its peak to trough and call it “bounce back effect.” Balke and Wynne (1996) examine postwar industrial production of the G7 countries and find that the bounce-back effect is true for the remaining six developed countries as well. Sichel (1994) finds evidence of three phases in US business cycles: contraction, high-growth recovery, and a moderated-growth period following recovery. Beaudry and Koop (1993) find that the negative shocks that cause contraction in US output contain important transitory component thus they follow by a phase called “reversion.” Kim and Murray (2002) find that 77% to 96% of the observed recessionary variance of the US coincident series is due to the common transitory component, suggesting that the largest part of negative shocks over the business cycle is temporary and business cycles have three phases: recession, partial recovery, and normal growth.

I find that international business cycle is characterized by two phases: a high-growth phase and a low-growth phase. The high-growth and low-growth phases of international business cycle are qualitatively different from US business cycle regimes estimated by Hamilton (1989). The high- and low-growth states of the international trend can not be interpreted as classical expansion and recession phases of business cycles because they are both positive and are highly persistent. The expected duration of the high-growth regime for the common permanent component of G7 countries is $(1-q)^{-1}$ and is approximately 17 years. Accordingly the expected duration of the low-growth regime is $(1-p)^{-1}$ about 82 years. The transition probabilities imply that unconditional probability of being in high-growth regime is $(1-q)/(2-p-q)=0.827$ around 83% and in the low-growth regime is accordingly 17%. In addition, the probabilities of switching from one regime to another $1-q$ and $1-p$ are very low.

I conclude that international business cycles lack the recession and partial recovery phases. However the phase of permanent growth in the individual countries transferred into two phase process with high-growth regime and low-growth regime.

Result 2:

The high persistence of both the high-growth and low-growth regimes results in a one time switch from the high-growth regime to the low-growth regime. As it is illustrated in Figures 3 and 4, the switch of the international trend from the high-growth regime to a low-growth regime occurred in 1973. The outcome is in accord with the historically known post WWII high growth period and a slow down in the growth in early 1970s. Khan and Rich (2003), who examined the productivity changes in US based on growth theory, estimated regime-switching dynamic factor model of Kim and Murray (2002) for non-farm sector output, labor productivity, real compensation per hour and hours worked. They find that US productivity trend switches from a high-growth regime to a low-growth regime in 1973 and switches back to high-growth regime in 1996. The results obtained by Khan and Rich (2003) strengthens the conclusion that the estimated switch to low-growth in 1973 is indeed the international negative shock that was the cause of productivity slowdown in the developed countries.

The “early 1970s productivity slow-down” phenomenon has been extensively examined in the economic literature. Different empirical and theoretical explanations were given in this regard. Nordhaus (1982) lists capital stock, labor, energy, regulations, R&D, and sectoral shifts as “best guess” sources of the productivity slowdown. Shigehara (1992) highlights the first oil price increase, R&D slowdown, entrance of inexperienced workers to the labor market and the breakdown of the Bretton Woods as most commonly addressed factors as explanations for the slowdown. While some of these explanations are not common shocks to all G7 countries, for example R&D slowdown, change in capital stock or quality of labor force, two explanations stand out – sudden increase in oil price and the collapse of the Bretton Woods system. Both of these events happened in 1973. However, considering the fact that the collapse of the Bretton

Woods system traces back to 1971 the oil price shock is the only international event that coincides with the estimated regime switch in the international trend.

Result 3:

The magnitude of the common positive shock to the permanent component for each of the countries is measured by $\gamma_i\mu_0$. Similarly the common negative shock for each of the countries is equal to $\gamma_i\mu_1$. The effect of international permanent shocks on each of the countries growth rates is illustrated in Table 3.

The country that is the most sensitive to the international shocks is Japan with $\gamma_5 \approx 0.37$, which is 0.71% annual decline in the growth during low-growth period and 1.58% annual increase in the growth during the high-growth period. The annual decline of the growth due to the common shocks for the other countries varies from 0.23% to 0.37% and the annual increase in growth for those countries varies from 0.13% to 0.21%. The least affected country is Germany with $\gamma_5 \approx 0.12$. Common positive shock to the permanent component of Germany will result in 0.51% annual increase in growth and the common negative shock will result in 0.23% annual decrease in the growth. This result is consistent with Nordhaus (1982) who examines productivity and output growth for G7 countries' for 1960 to 1978 time period. According to his estimates of productivity slowdown "Japan showed the largest slowdown, 5.2 percentage point deceleration, while Germany's is the smallest, slowing 1.2 percentage points".

5. Conclusion

In this study I employ a multivariate dynamic factor model with four-state Markov regime-switching to measure permanent and transitory components of international business cycles of G7 countries. I define the cyclical component as the stationary deviations from stochastic trend. The implemented model has several advantages over the empirical research that has been conducted in the field of

international business cycles. It is clear in its definition of international cycle and trend, and does not require prior detrending of time series.

I find that the dynamics of international permanent and transitory components is different from the dynamics of US business cycle fluctuations. The studies of the US permanent and transitory components show that US business cycles are well characterized by three phases: recession, recovery²⁵ and normal growth. It was also shown that the negative shocks to the US economy are mostly transitory. I did not find any evidence of statistically significant international transitory component. I find that the negative shocks to international business cycle are permanent and that the international business cycle has only two phases: a high-growth phase and a low-growth phase.

My results suggest that the international business cycle do not exhibit any cyclical fluctuations but undergo a long-run trend change from a high-growth state to a low-growth state. From the theoretical perspective, this implies that theories which explore the affects of permanent shocks are more appropriate for the study of international business cycles than theories that explore the propagation of transitory shocks. It also emphasizes the importance of growth theories within an international business cycles framework.

According to my results, the international trend component is in a high growth phase from 1961 to 1973. In the second quarter of 1973, the international trend switches from a high-growth regime to a low-growth regime. Both regimes are very persistent and the probability of switching from the low-growth regime back to the high-growth-regime is in the vicinity of zero. The timing of the switch corresponds to the well know phenomenon. There have been numerous reasons discussed in the economic literature on what caused the productivity slow down. We can learn from this study that there was a common to all the countries international shock that influenced the growth rate of each country. I estimated the fraction of each country's growth rate decline which is due to the international shock of 1973. I also find that Japan's real output is the most sensitive to the international permanent shocks and Germany's output is the least sensitive.

²⁵ Within the US business cycle literature, the recovery phase is referred to as revival, high-growth period, "bounce back" effect, peak-reversion or partial peak-reversion.

There is still more to be done in analyzing permanent and transitory components of international business cycles. One of the possible extensions is to model different groupings of the countries. For example, do underdeveloped countries' international business cycles share the same characteristics as the international business cycle of developed countries? Another possible extension to this study is to include more series for each of the countries and to measure the relative importance of country specific trend and cycle to international trend and cycle.

Table 1: Maximum Likelihood Estimates (Full Model): Quarterly GDP, 1960:1 to 2002:2, $i = \text{Canada, France, Germany, Italy, Japan, UK and US}$

Transition Probabilities			
q_1	0.9897 (0.0130)	p_1	0.9949 (0.0063)
q_2	0.9779 (0.0311)	p_2	0.4617 (0.5773)
Regime Dependent Parameters			
μ_0	0.8699 (0.2497)	μ_1	-0.3968 (0.1373)
λ	-6.7408 (6.2932)		
Permanent Factor Loadings			
γ_1	0.2410 (0.0588)	γ_2	0.2313 (0.0426)
γ_3	0.1328 (0.0456)	γ_4	0.2156 (0.0549)
γ_5	0.3435 (0.0544)	γ_6	0.1626 (0.0512)
γ_7	0.1921 (0.0568)		
Transitory Factor Loadings			
α_1	-0.0204 (0.0122)	α_2	0.1689 (0.0399)
α_3	0.0094 (0.0163)	α_4	0.0133 (0.0162)
α_5	-0.0095 (0.0131)	α_6	0.0372 (0.0165)
α_7	-0.0103 (0.0121)		
Autoregressive Parameters for the Common and Idiosyncratic Components			
ϕ^r	0.5583 (0.1011)	ϕ^c	-0.0888 (0.0467)
ϕ_1	0.1850 (0.0802)	ϕ_2	0.5655 (0.1887)
ϕ_3	-0.1640 (0.0799)	ϕ_4	-0.1655 (0.0820)
ϕ_5	0.0796 (0.1268)	ϕ_6	-0.0522 (0.0791)
ϕ_7	0.2073 (0.0764)		
Idiosyncratic Innovation Standard Deviations			
σ_1	0.8567 (0.0503)	σ_2	0.1542 (0.0599)
σ_3	0.9553 (0.0537)	σ_4	0.9140 (0.0530)
σ_5	0.7677 (0.0542)	σ_6	0.9298 (0.0521)
σ_7	0.8957 (0.0508)	$\sigma_{u,1}$	14.9558 (4.7181)
ln L = -521.1546			

Standard errors are in parentheses.

Table 2: Maximum Likelihood Estimates for Common Trend: Quarterly GDP, 1960:1 to 2002:2, $i =$ Canada, France, Germany, Italy, Japan, UK and US

Transition Probabilities			
q	0.9855 (0.0165)	p	0.9970 (0.0046)
Regime Dependent Parameters			
μ_0^w	1.0720 (0.4758)	μ_1^w	-0.4821 (0.1846)
Permanent Factor Loadings			
γ_1	0.1859 (0.0570)	γ_2	0.1915 (0.0410)
γ_3	0.1186 (0.0429)	γ_4	0.1633 (0.0459)
γ_5	0.3690 (0.0658)	γ_6	0.1332 (0.0482)
γ_7	0.1604 (0.0555)		
Autoregressive Parameters for the Common and Idiosyncratic Components			
ϕ^r	0.5364 (0.1743)		
ϕ_1	0.2069 (0.0796)	ϕ_2	-0.4735 (0.0708)
ϕ_3	-0.1526 (0.0774)	ϕ_4	-0.1289 (0.0710)
ϕ_5	-0.1261 (0.1135)	ϕ_6	-0.0773 (0.0808)
ϕ_7	0.2215 (0.0767)		
Idiosyncratic Innovation Standard Deviations			
σ_1	0.8878 (0.0511)	σ_2	0.8251 (0.0485)
σ_3	0.9581 (0.0538)	σ_4	0.9466 (0.0558)
σ_5	0.6771 (0.0561)	σ_6	0.9588 (0.0558)
σ_7	0.9059 (0.0510)		
ln L = -632.3428			

Standard errors are in parentheses.

Table 3: The Effect of International Permanent Component on G7 Countries

	High-Growth Regime	Annual Percents	Low-Growth Regime	Annual Percents
Canada	0.20	0.80	-0.09	-0.36
France	0.21	0.82	-0.09	-0.37
Germany	0.13	0.51	-0.06	-0.23
Italy	0.18	0.70	-0.08	-0.31
Japan	0.40	1.58	-0.18	-0.71
UK	0.14	0.57	-0.06	-0.26
US	0.17	0.69	-0.08	-0.31

Figure 1: Common Cycle for G7 Countries: Model (1)-(5)

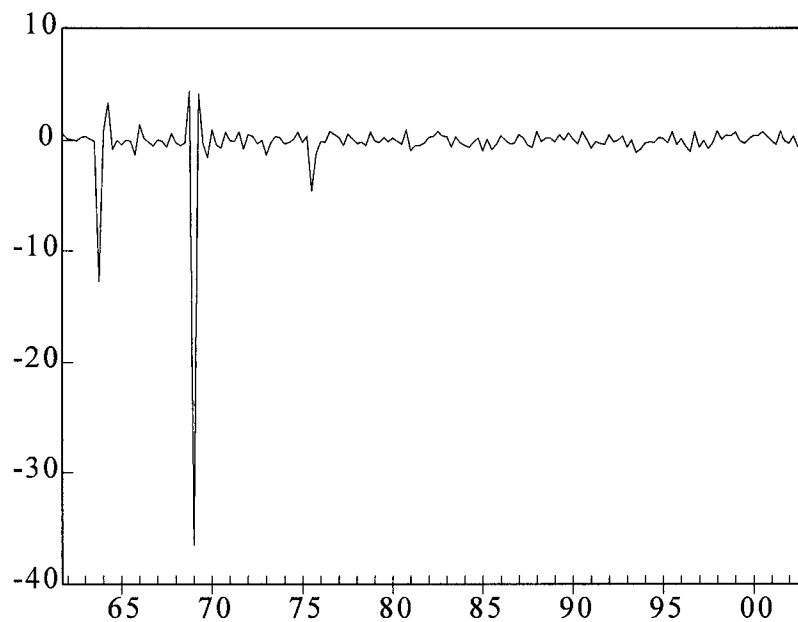


Figure 2: Smoothed Probability of Contraction for Common Cycle: Model (1)-(5)

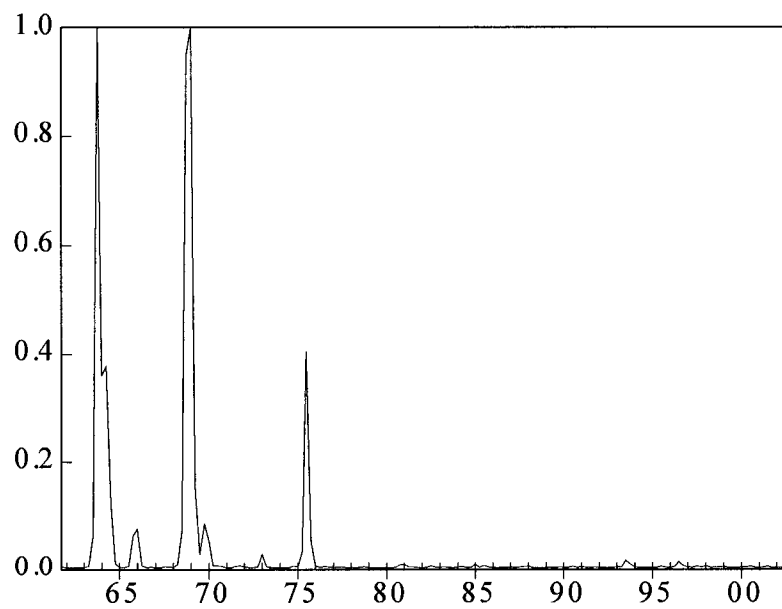


Figure 3: Common Trend for G7 Countries: Model (1')-(4')

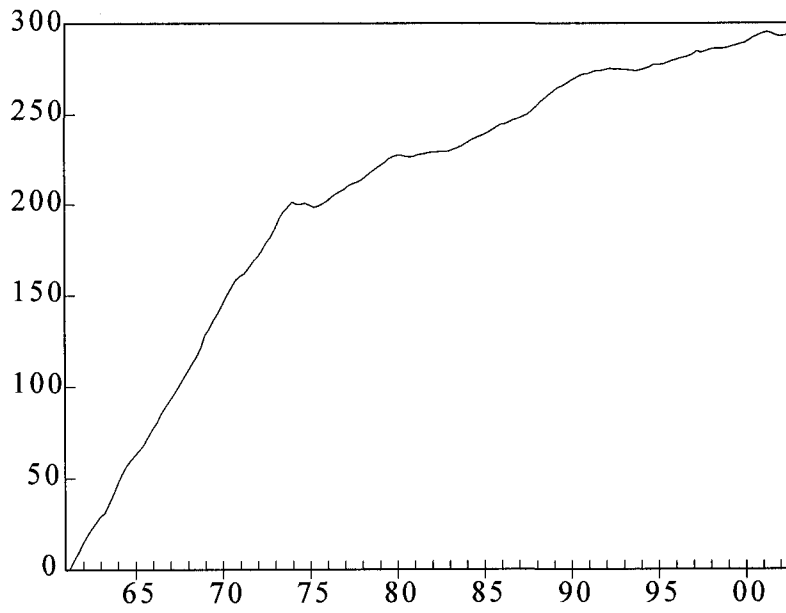
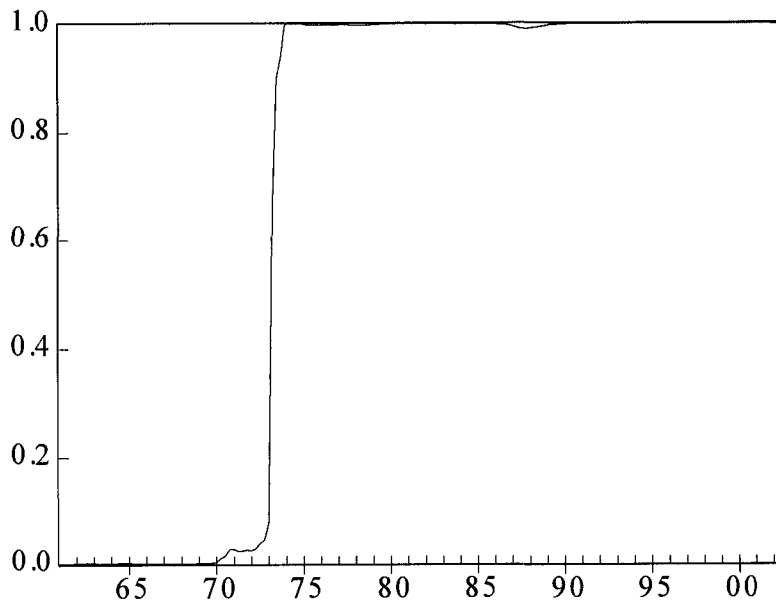


Figure 4: Smoothed Probability of Contraction for Common Trend: Model (1')-(4')



Study III: Driving Forces Behind International Business Cycle Fluctuations: Can One Identify Them?

1. Introduction

The comovement of economic time series has played an important role in the analysis and forecasting of business cycle fluctuations. As early as Mitchell (1927), Mitchell and Burns (1938), and Burns and Mitchell (1946), research on the subject has emphasized the comovement of aggregate variables of one country as well as the comovement of major macroeconomic variables across countries as significant empirical facts of business cycle fluctuations. The increase of financial integration, diminishment of trade barriers, as well as the creation of single currency unions have promoted the current field of research on the comovement of main economic indicators across countries, tagged as international business cycles.

The objective of this study is an empirical investigation of real GDP series of G7 countries with the purpose of identifying the transmission of permanent shocks from one country to the others. This investigation becomes possible with an application of cointegrated VAR methodology which consists of several types of misspecification and identification tests as suggested by Juselius (2005). The application of this methodology is a new approach to the empirical investigation of international business cycles. It is an attempt to find a model that best illustrates the driving forces behind G7 business cycles. Consequently, it is an exercise in structuring the long-run information about driving forces in the data via econometric identification restrictions on common long-run relations and common stochastic trends of the system. The methodology presented here identifies the pushing countries within the system, as well as the adjustment mechanism towards the steady-state. The period of normal economic growth is considered the steady-state for international business cycles.

The main result of the study is that the US affects the stochastic path of the remaining six countries in the system, while it is not influenced by them. Shocks to the UK have only a transitory effect on the countries in the system. Thus, the UK does not exhibit any permanent influence on any of the countries. Germany and Japan have permanent effects on each other and are influenced by shocks from France and the US. All the countries but Japan and the UK have a permanent influence on Italy's economy while Italy does not influence any of the countries in the system. The stochastic paths of Canada, France and UK are influenced only by US permanent shocks.

The contribution of this study is that it is the first study to analyze the transmission of permanent shocks between the seven most developed countries of the world. The substantial literature on international business cycles that applies VAR methodology is based on the analysis of previously detrended time series, thus it is unable to demonstrate the importance of permanent shocks. Examples of such studies are Stock and Watson (2003) and Bordo and Helbling (2003). While Bordo and Helbling (2003) find it difficult to distinguish between "true" global shocks and shocks that originated in major countries, they find that "shocks in the largest [G7] country, the US, where unsurprisingly, a key factor in the worldwide Great Depression." The key result of this study is that the US continues to be the key factor that permanently affects the economic fluctuations of other countries. One of the major results of Stock and Watson (2003) is the emergence of two groups among G7 countries: the English-speaking group (Canada, the UK and the US) and the Euro-zone countries (France, Germany and Italy). This study does not find any evidence of increased synchronization within these two groups. No long-run, steady-state adjusting relation was identified between France, Germany and Italy, while the long-run relation between Canada, the UK and the US was not significant enough to be considered.

Support for the US as the main driving force behind G7 countries can be found in Filardo and Gordon (1994). Filardo and Gordon (1994) investigate international business cycles between Canada, the UK and the US. They apply Markov switching multivariate time series model to identify high- and low-growth states and to capture sources of comovement. The main result of their study is that "international business cycle phases

do not appear to evolve independently of each other, nor are they perfectly synchronized,” however US phases lead both Canadian and UK phases.

The rest of the study is organized as follows. Section 2 presents general background on the cointegrated VAR model, trend and cycle decomposition within the cointegrated VAR model, as well as a determination of the number of common trends in the model. Section 3 presents the properties of the time series examined and finds the appropriate VAR model that best fits the examined data. Section 4 discusses the empirical results, and section 5 concludes.

2. Unrestricted VAR Model Specification and Determination of Cointegration Rank²⁶

The general form of the k^{th} order vector autoregressive model, VAR(k) is:

$$x_{it} = \Pi_1 x_{it-1} + \dots + \Pi_k x_{it-k} + \Phi D_t + \varepsilon_{it}, \quad t = 1, \dots, T \quad (1)$$

where $x_{it} = x_{1t}, x_{2t}, \dots, x_{pt}$ are the variables of interest, D_t is the deterministic term, which contains permanent impulse dummies and the constant of the model, $\Pi_1, \dots, \Pi_k, \Phi$ are parameters of the model, $\varepsilon_{it} \sim iidN(0, \Omega)$ and k is the lag length. The corresponding vector error correction model (ECM) is:

$$\Delta x_{it} = \Gamma_1^{(m)} \Delta x_{pt-1} + \Gamma_2^{(m)} \Delta x_{pt-2} + \dots + \Gamma_{k-1}^{(m)} \Delta x_{pt-k+1} + \Pi x_{t-m} + \Phi D_t + \varepsilon_{it} \quad (2)$$

where $\Pi = -I + \sum_{j=1}^k \Pi_j$, m is an integer between 1 and k , Π matrix summarizes the long run effects in the system and stays unchanged regardless the chosen lag m and $\Gamma_i^{(m)}$ for $i = 1, \dots, k-1$ contains short-run effects of the model and depends on the chosen lag m within the model.

$\Pi = \alpha\beta'$ decomposition lets us identify the adjustment mechanism in the system examined. Assuming that r is the cointegration rank in the model and that p is the

²⁶ This section relies on Juselius (2005) and Søren Johansen's lecture notes for summer school on cointegrated VAR models.

number of variables in the model, β' is described by an $r \times p$ matrix, where $\beta' x_{it}$ is the derivation of each variable i from the steady state of the system, and α is a $p \times r$ matrix that shows the speed of adjustment to the steady state for each of the variables in the system. β' represents the common long run relations in the system with corresponding α factor loadings (Figure 2).

According to the Granger representation theorem stated in Engle and Granger (1987) and Johansen (1991) under the assumptions that the variables are integrated of first order $x_{it} \sim I(1)$, and that Δx_{it} and $\beta' x_{it}$ have stationary and invertible vector autoregressive moving average representation, the ECM

$$\Delta x_{it} = \alpha \beta' x_{it-1} + \sum_j^{k-1} \Gamma_j \Delta x_{it-j} + \Phi D_t + \varepsilon_{it}$$

can be presented in its moving average form as follows:

$$x_{it} = C \sum_{s=1}^t \varepsilon_{is} + C(L) \varepsilon_{it} + \tau(t) + A \quad (3)$$

where $C = \beta_{\perp} (\alpha'_{\perp} \Gamma \beta_{\perp})^{-1} \alpha'_{\perp}$, $\Gamma = I_p - \sum_{j=1}^{k-1} \Gamma_j$, $\tau(t) = C \Phi \sum_{s=1}^t D_s + C(L) \Phi D_t$, the coefficients of $C(L)$ are given by $\Delta C_i = \Pi C_{i-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta C_{i-j}$ for $i=1,2,\dots$, and A depends on the initial values of x_{it} . The Granger representation theorem presents a trend-cycle decomposition of cointegrated VAR such that $\alpha'_{\perp} \sum_{s=1}^t \varepsilon_{is}$ are common stochastic trends of the model corresponding to $\tilde{\beta}_{\perp} = \beta_{\perp} (\alpha'_{\perp} \Gamma \beta_{\perp})^{-1}$ factor loadings and $C(L) \varepsilon_{it}$ is the stationary process, which is cycle. The number of common trends is equal to $p-r$.

$\alpha'_{\perp} \sum_{s=1}^t \varepsilon_{is}$ common trends, derived from the moving average representation of the model, are the pushing forces of the system that push the process along the attractor set. At the same time, the process is pulled towards the attractor set by the adjustment coefficients of autoregressive representation of the model. Figure 2 presents an illustration of pushing and pulling forces of the two dimensional system $x_{it} = (x_{1t}, x_{2t})$ along the steady-state position. Assuming that the steady-state corresponds to the

$\beta' = [I, -I]$ cointegration relation, the attractor set $\beta_{\perp} = [I, I]$ will correspond to a 45° degree line along which $x_{1t} = x_{2t}$ and the system is in the steady-state.

The reduced model of ECM is derived by application of the Frisch-Waugh theorem. To derive the reduced model the equation (2) is written in the more compact form:

$$Z_{0t} = \alpha\beta' Z_{1t} + \Psi Z_{2t} + \varepsilon_{it} \quad (4)$$

where Z_{0t}, Z_{1t}, Z_{2t} and Ψ are defined as:

$$\begin{aligned} Z_{0t} &= \Delta x_{it} \\ Z_{1t} &= x_{it-1} \\ Z_{2t} &= [\Delta x_{it-1}, \Delta x_{it-2}, \dots, \Delta x_{it-k+1}, D_t] \\ \Psi &= [\Gamma_1, \Gamma_2, \dots, \Gamma_{k-1}, \Phi] \end{aligned}$$

Then, applying the Frisch-Waugh theorem to (4), the reduced form (5) is obtained:

$$R_{0t} = \alpha\beta' R_{1t} + R_{\varepsilon t} \quad (5)$$

where R_{0t} and R_{1t} are defined from the auxiliary regressions:

$$\begin{aligned} Z_{0t} &= \hat{B}_1' Z_{2t} + R_{0t} \\ Z_{1t} &= \hat{B}_2' Z_{2t} + R_{1t} \end{aligned}$$

$\hat{B}_1' = M_{02} M_{22}^{-1}$, $\hat{B}_2' = M_{12} M_{22}^{-1}$ are OLS estimates, $M_{ij} = \sum_t (Z_{it} Z_{jt}') / T$, $p \lim_{T \rightarrow \infty} M_{ij} = \Sigma$ and $R_{\varepsilon t}$ is the error term. The reduced form (5) is referred to as the concentrated model of cointegrated VAR because it concentrates out the short-run transitory effects and attains a more apparent long-run adjustment model.

Determination of cointegration (Johansen LR trace test, Johansen (1988, 1991, 1994)) is based on the concentrated model (5). It tests the null of the $p - r^*$ unit root processes in the model against the alternative of $r^* = p$, corresponding to no unit roots and stationary x_t ((H_{r^*} / H_p)). The trace test has a nested character, such as:

$$H_0 \subset H_1 \subset \dots \subset H_{r^*} \subset \dots \subset H_p.$$

It starts with the most restrictive case of having no cointegration relations ($H_{r=0} / H_p$), thereby having the maximum number of unit roots, and works its way down to having no unit roots ($H_{r=p} / H_p$). The LR test is:

$$-2 \ln LR(H_r / H_p) = -T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i)$$

where λ_i s are the eigenvalues of the system defined as $|\lambda S_{11} - S_{10} S_{01}^{-1} S_{01}| = 0$ for $\hat{\lambda}_1 > \hat{\lambda}_2 > \dots > \hat{\lambda}_p$. The likelihood values for H_r and H_p are given by:

$$L_{\max}^{-2/T}(H_r) = |S_{00}| \prod_{i=1}^r (1 - \hat{\lambda}_i)$$

$$L_{\max}^{-2/T}(H_p) = |S_{00}| \prod_{i=1}^p (1 - \hat{\lambda}_i)$$

and

$$S_{ij} = T^{-1} \sum_{t=1}^T R_{it} R'_{jt}$$

$\hat{\lambda}_1 = 0$ corresponds to zero cointegration relations in the model. The unit root corresponds to zero eigenvalues $\hat{\lambda}_i$.

It is important to note that the trace test has size and power distortion, in particular for small sample size time series and close to unit root processes. To correct for the size of the test, small sample Barlett correction can be applied, developed by Johansen (2002). However, it does not solve the power problem. Juselius (2005) mentions that there are cases when for the close to the unit circle hypotheses, the size of the test and the power of the relevant alternative are almost of the same magnitude. Thus, Juselius (2005) suggests using additional information to determine the cointegration rank, such as: recursive graphs of the trace statistic, roots of the companion matrix, t-values of the α coefficients, and graphs of the cointegration relations. Recursive graphs of the trace statistics are calculated by $-T_j \ln(1 - \lambda_i)$, $j = T_1, \dots, T$. They will grow linearly for $\lambda_i \neq 0$, all $i = 1, \dots, r$ and stay constant for $i = r + 1, \dots, p$. The largest characteristic root of the companion matrix will be close to the unit circle if the $r^{\text{th}} + 1$ cointegration vector is nonstationary and is incorrectly included in the model. Similarly, we will not gain much

information from including the $r^{th} + 1$ cointegration vector if the t-values corresponding to the vector α coefficients are not significant. We should also reconsider the choice of r if the graph of the r^{th} cointegration relation has nonstationary behavior.

3. Data Analysis within VAR model

The data examined in this study consist of quarterly real GDP series for G7 countries: Canada, France, Germany, Italy, Japan, the UK and the US. They cover the period from 1971:1 to 2002:4, are seasonally adjusted, and have a base year of 2000. The data are taken from the OECD Quarterly National Accounts database and the Datastream database. The GDP series of France was available only from 1978. Consistent time series of real GDP for Japan is available from 1980. The missing observations at the beginning of the examined period for France and Japan were constructed using GDP volumes of the countries following the methodology suggested by Stock and Watson (2003).

The plots of log GDPs of G7 countries in levels and 1st differences as well as their residuals are presented in the Figures 1a-1g. The differenced series show some presence of innovative outliers. The figures of standardized residuals show that the residuals of the model estimated are neither independent nor normally distributed. Based on the standard deviations there are several innovative outliers, which correspond to extraordinary large shocks due to economic interventions or changes in the economy's regime, with delayed dynamic effect in the data. Those outliers are identified as standardized residuals larger than $(1 - 0.025)^{1/T}$, where T is the number of observations in each series. In this case T is equal to 135 and $(1 - 0.025)^{1/T} = 3.56$

Estimation of a VAR model is based on the assumption that variables' residuals follow a white noise process. Extraordinary large shocks that correspond to an economic reform or intervention cause a violation of the normality assumption. The deviation from the normality assumption leads to incorrect statistical inferences. Thus it is important to

identify the dates of such shocks and to correct them with unrestricted permanent impulse dummies, where each of the impulse dummies is of the form $[0 \dots 0 \ 1 \ 0 \dots 0]$.

Prior to correcting for innovative outliers, the VAR(2) model is estimated, and several specification tests are performed to check for residual autocorrelation and normal distribution. The estimated VAR(2) model contains an unrestricted mean, which allows for a linear trend in the data but not for one in the cointegrated VAR system. The number of variables, $p=7$, $x'_{it} = [x_{1t} \ x_{2t} \ x_{3t} \ x_{4t} \ x_{5t} \ x_{6t} \ x_{7t}]$ corresponding to the natural logarithms of the real GDP series of Canada, France, Germany, Italy, Japan, the UK and the US. The GDP series are assumed to follow a random walk with drift process. The test results are presented in Table 1a. LM_1 and LM_4 tests test of ε_{it} autocorrelation with ε_{it-1} and ε_{it-4} respectively test the null of no autocorrelation. The LM-test is calculated using a Wilks' ratio test with a small sample correction (Anderson (1984)). It is asymptotically distributed χ^2 with p^2 degrees of freedom.

$$LM(j) = -\left(T - p(k+1) - \frac{1}{2}\right) \ln \left(\frac{|\tilde{\Omega}(j)|}{|\hat{\Omega}|} \right)$$

Based on the result of the LM_1 test we reject autocorrelation in the first lag of residuals. The Multivariate test for normality is based on Doornik and Hansen (1994). It is distributed χ^2 with $2p$ degrees of freedom. The null of the test is normality and it is rejected with a zero p-value. Trace correlation is a joint measure of explained variation in the VAR model and is equal to 0.22.

$$\text{Trace correlation} = 1 - \text{trace}(\hat{\Omega} \text{Var}(x_t)^{-1}) / p$$

The Next section of Table 1a presents univariate properties of time series. Residual heteroskedasticity (ARCH) test statistics are calculated as follows:

$$(T - k) \times R^2$$

where R^2 is from the auxiliary regression $\varepsilon_{it}^2 = \gamma_0 + \sum_{j=1}^k \gamma_j \varepsilon_{it-j}^2 + u_{it}$ and k is the number of lags. The Univariate Jarque-Bera test for normality is also calculated:

$$\frac{T(\text{skewness})^2}{6} + \frac{T(\text{kurtosis} - 3)^2}{24}$$

Both tests are asymptotically distributed χ^2 with 2 degrees of freedom. Under the null of the Jarque-Bera test, the errors are normally distributed:

$$\left(\frac{\hat{\varepsilon}_{it}^2}{\hat{\sigma}_i} \right)^3 \stackrel{a}{\sim} N(0,6), \quad \left(\frac{\hat{\varepsilon}_{it}^2}{\hat{\sigma}_i} \right)^4 \stackrel{a}{\sim} N(3,24)$$

Based on the univariate and multivariate residual analyses, there are significant deviations from normal distribution in skewness and/or kurtosis of Germany, Japan, the UK and the US. The null of no residual heteroskedasticity is rejected for Canada, the UK and the US, and the null of normal distribution of each of the time series is rejected for Germany, Japan, the UK and the US. Univariate analyses demonstrate that Germany, Japan, the UK and the US time series are the potential cause for rejection of the multivariate normality test. Thus several dummy variables have to be introduced to correct for innovative outliers existing in those series.

Based on VAR(2) standard residuals output the following outliers $\left(\left| \hat{\varepsilon}_{it} \right| > 3.56 \hat{\sigma}_\varepsilon \right)$ were identified in the time series of G7 countries: Germany 1987:1 (standardized residual = -4.09) and 1991:1 (standardized residual = 8.20); Japan 1974:1 (standardized residual = -4.83), the UK 1973:1 (standardized residual = 4.41) and 1979:2 (standardized residual = 4.15); and the US 1978:2 (standardized residual = 3.77). The countries that were found to have outliers are consistent with prior examination of the specification tests.

The UK series contains an outlier in the 1973 first quarter that is a result of the first oil price shock in the world economy. The effect of the oil shock can possibly explain the outliers for the first quarter of 1974 in Japan GDP. The volatility of the GDP series in the early 1970s can be also explained with the gradual collapse of the Bretton Wood system and the change from fixed to flexible exchange rates. The shocks in the US and UK GDP in the late 70s are connected with the second oil crisis. The German series contains an obvious outlier in the first quarter of 1991 as a result of German unification. The dates that are identified as innovative outliers are consistent with the

turning points dates estimated by Krozlig (2003). The 1973:1, 1974:1, 1974:4, 1978:2, 1979:2, 1987:1 and 1991:1 outliers were also identified as innovative outliers by the Nielsen (2004) program.

Table 1b presents residual analysis of VAR(2) that incorporates D_t regressor with the following permanent impulse dummy variables identified in the data:

$$D_t' = [D_{p731} \quad D_{p741} \quad D_{p782} \quad D_{p792} \quad D_{p871} \quad D_{p911}].$$

According to LM_1 and LM_4 tests the null of no autocorrelation between the residuals is accepted at the 7 and 12 per cent levels respectively. The LM test of normality of the VAR model is only borderline rejected. There is substantial skewness and/or kurtosis discrepancy from normal distribution for France, the UK and the US. However it should not cause problems for the estimation of VAR model since the univariate normality test fails to reject normal distribution in all seven series. According to the ARCH test we are unable to reject heteroskedasticity in the residuals of the US and Canada. These problems can potentially arise when there are several standardized residuals with an absolute value close to 3.56, which can create a heteroskedastic effect. Such behavior is noticed on the plot of standardized residuals of Canada and the US (Figures 1a and 1f) in the time periods of 1971 to 1973 and 1979 to 1982 respectively. The low p-value of multivariate LM normality test in the system can be attributed to the outcome of the heteroskedasticity test. Nonetheless, cointegration results are robust to small ARCH and excess kurtosis (Gonzalo, 1994).

The next important step in the selection of the VAR model that best fits the examined time series analysis is to determine the correct lag length. For this purpose Schwartz, Akaike and Hannan-Quinn information criteria tests were conducted. The tests are defined by:

$$SIC = \ln \left| \hat{\Omega}^k \right| + (p^2 k) \frac{\ln T}{T}$$

$$AIC = \ln \left| \hat{\Omega}^k \right| + (p^2 k) \frac{2}{T}$$

$$HQ = \ln \left| \hat{\Omega}^k \right| + (p^2 k) \frac{2 \ln \ln T}{T}$$

Lag selection tests along with $\ln|\hat{\Omega}| = -2/T(\ln L_{\max})$ are reported in Table 2. In calculation of SIC, AIC and HQ, the number of observations is kept constant for all lags. Schwartz and Hannan-Quinn criteria suggest lag 1, while AIC suggests lag 2. However, based on the p-values of LM₁ test and normality test, as well as the measure of trace correlation, it is obvious that lag 2 is a better choice to model the time series of interest.

The VAR(2) with six unrestricted dummies and unrestricted constant is:

$$x_{it} = \Pi_1 x_{it-1} + \Pi_2 x_{it-2} + \Phi D_t + \varepsilon_{it}, \quad t = 1, \dots, T$$

It corresponds to VECM:

$$\Delta x_{it} = \Gamma_1^m \Delta x_{it-m} + \Pi x_{it-m} + \Phi D_t + \varepsilon_{it},^{27} \quad \varepsilon_{it} \sim iidN(0, \Omega)$$

The results of the LR trace test are presented in Table 3. The Trace test statistics fail to reject the hypotheses of $p-r=6$ common trends and $r=1$ cointegration relations. Considering the fact that this hypothesis is only borderline failed to reject with $p-value = 0.057$, and the size and power distortions of the trace test, the next alternative of $p-r=5$ trends with $r=2$ cointegration relations is more accurate. The reported eigenvalues indicate that even $r=3$ can be plausible. Inspection of the roots of the companion matrix (Table 4) illustrate that indeed the lowest root corresponds to $r=1$ and equals to 0.58 . Nevertheless, the largest roots for $r=2,3,4$ are equal to 0.92 and 0.93 and correspond to stationary yet slower equilibrium adjusting processes. Within the international business cycles framework, the steady state is interpreted as normal growth and the adjustment to the steady state can be quite slow. Thus the root of 0.93 is still suitable for the purpose of this study. The Figure 3 illustrates recursively estimated traces. Based on the trace of the concentrated model there are at least two, and possibly 3 linearly growing traces.

There is no clear indication of the number of common trends in this model. Therefore further long-run identification of this study is based on 3 different models with $r=1$, $r=2$ and $r=3$ marked as Model 1, Model 2 and Model 3 respectively.

²⁷ Note that for VAR(2) the only choices for m is $m=1,2$. In this regard, $\Gamma_1^{(1)} = -\Pi_2$ or $\Gamma_1^{(2)} = (I - \Pi_1)$, while matrix Π will remain unchanged.

4. Identification of Long-Run Structure and Common Trends

Information on the long-run relations of the model is concentrated in the β matrix since it carries information on the countries that are cointegrated. It also characterizes pulling to the steady state forces of each country. In this regard, the long-run identification problem translates into the identification of a β matrix that will explain the best the relationship between the countries' business cycles. This objective is achieved by posing testable restrictions on vectors in the β matrix and comparing the significance of the tests. In order for the restrictions on β to be testable, the restrictions should be over-identifying, which means that the number of the restrictions on each vector β_i should be bigger than $r - 1$. Long run restrictions on β are tested with the LR test. The technical derivation of the test procedures is shown in Johansen and Juselius (1992).

Equally important is to identify a structure that characterizes the α matrix the best since the structure of common trends is defined as $\alpha' \sum_{s=1}^t \varepsilon_{is}$ and depends on α . There are two important properties of α that should be tested for: weak exogeneity and a known vector. The weak exogeneity condition is tested for by the LR test described in Johansen and Juselius (1990). The hypothesis of zero row restriction in α tests whether the cumulative residuals of the variable (corresponding to the zero row) are a common driving trend in the system. If the hypothesis is accepted, that variable affects the long run stochastic path of the other variables while at the same time is not affected by them. A known vector in α is tested for with the LR test described in Johansen (1996). It tests whether one of the variables is exclusively adjusting to one cointegrating relation, while the other variables are exclusively adjusting to the remaining cointegration relations.

Table 5 presents the results from different restrictions posed on the β matrix. Restrictions that were rejected are not presented in the table. Constrained $\beta^c = (\beta_1^c, \dots, \beta_r) = (H_1 \varphi_1, \beta_2, \dots, \beta_r)$, where H_1 is a restriction design matrix and φ_1 is a coefficient matrix. In the 0-1 notation of the Table 5, 0's correspond to the variables whose coefficients in the β_i vector are restricted to zero and 1's correspond to the

variables in the vector that are cointegrated. For the purpose of simplicity, the coefficients of the restricted vectors are suppressed. The remaining $r - 1$ vectors in β assumed known and are not restricted. In Model 1 the cointegration rank equals to one and thus there are no known vectors.

In Model 1, the first hypothesis of a cointegration relation between Germany, Italy, Japan, the UK and the US is borderline failed to reject. $H_{1,2}$ has higher p-value once the US is dropped from the relation and Canada is added. However the $H_{1,3}$ hypothesis is the most explanatory one, since the p-value is the highest of the three relations accepted. The higher p-value is due to the addition of France to the previous relation. Thus the $H_{1,3}$ hypothesis is chosen as the most characteristic relation for Model 1.

Hypotheses $H_{2,1}$ to $H_{2,8}$ test cointegration relations between two countries. We borderline accept stationary relations between the following pairs: France and the UK, France and the US, Germany and the US, Japan and the UK, Japan and the US, and the UK and US. The cointegration relations between Canada and France, and Germany and the UK are accepted with higher p-values (0.32 and 0.23 respectively). $H_{2,1}$ to $H_{2,9}$ cointegration relations are irreducible, meaning that they are not a combination of other stationary relations and will be rejected if one of the variables is dropped from the relation. Hypotheses $H_{2,10}$ to $H_{2,22}$ are built based on already uncovered irreducible relations and are tested to find out whether we can extract more information about the system if some irreducible relations are combined. $H_{2,10}$ to $H_{2,13}$ describe the cointegration relations between 3 countries, with the highest p-value in $H_{2,13}$ between France, Germany and US. $H_{2,14}$ to $H_{2,17}$ describe the cointegration relations between 4 countries. The relations $H_{2,14}$ and $H_{2,17}$ have the highest p-values. $H_{2,18}$ to $H_{2,22}$ describe the cointegration relations between 5 countries, where $H_{2,22}$ has the highest p-value.

Table 6 presents joint tests on restricted β vectors that were accepted with high p-values. The restricted β matrix that has the highest explanatory power is the one that combines the Model 1 relation in the β_1 vector with the 3 country (France, Germany and the US) cointegration relation from $H_{2,13}$ in β_2 . In this regard Model 2(4) contains the necessary information from Model 1 and adds additional information about the system by including another stationary relation that has a p-value of 0.99. In addition, taking into consideration the fact that the choice of rank between $r = 1$ and $r = 2$ is based on the borderline p-value, Model 2(4) is chosen as a better representation of the time series of interest.

Table 6 also presents different combinations of Model 1 and Model 2 tested within Model 3. The three combinations with the highest p-values are presented. Model 3(1) is the most favorable one, since the $p\text{-value} = 0.99$ and since it includes relations from both Model 1 and Model 2.

Table 7 and Table 8 illustrate results from restrictions posed on the α matrix. Table 7 illustrates that regardless of the choice of rank, the US appears as a weakly exogenous variable. We fail to reject the hypotheses of Canada (Model 1-3) and Germany (Model 2) as weakly exogenous variables as well. However p-values for zero row restriction on the US have higher power and joint tests of the US and Canada\Germany decrease the p-value of the test, thus they do not have much explanatory power.

To check whether the US is indeed one of the common driving trends in the system we estimate a partial system conditioned on the US as a weakly exogenous variable. Table 9 shows cointegration rank test results for the partial system estimated.²⁸ In this system we fail to reject three $p - r = 3$ stochastic trends corresponding to the rank $r = 3$. This exercise helps us to shed light on the choice of rank for the full system. Comparing Tables 3 and 9, it is obvious that the Table 9 $r = 3$ choice (corresponding to

²⁸ Conditioning on weakly exogenous variables influences the tests of cointegration rank and the standard asymptotic tables are not valid (Harboe, Johansen and Rahbek, 1997). The critical values reported in Table 10 are from Harboe, Johansen and Rahbek (1997), generated under the assumption that the weakly exogenous variable in the system satisfies the condition of weakly exogeneity.

eigenvalues of 0.12 and 3 stochastic trends) is identical to the $r = 3$ choice of the Table 3 (eigenvalues are the same) and 4 stochastic trends, where one of the trends consists of cumulative residuals of the US only. Thus it estimates one trend more than the partial model with the US as a weakly exogenous variable. Therefore, preference in the full model should be given to the cointegration rank $r = 3$ alternative and $p - r = 4$ stochastic trends, where the row of the α matrix that corresponds to US should be restricted to zero. Consequently, further discussion of the restrictions on the α matrix and the final conclusions of the study are based on the already favorable Model 3(1). Joint restriction on β matrix and α matrix, corresponding to Model 3(1) and US as weak exogenous variable, is accepted with $p = 0.99$.

Table 8 shows the unit vector in α test results for seven countries. Test fails to reject the hypothesis for Germany and the UK with $p = 0.05$ and $p = 0.18$ respectively. However the joint test for those countries is rejected. The unrestricted α matrix from Model 3(1) presented in Table 11 illustrates that there are several coefficients that are not significant and can be restricted to zero, such as all three coefficients for the US (which is equivalent to weakly exogeneity), Germany's and Italy's coefficients in α_1 , Japan's coefficient in α_2 , and Canada's, France's, Germany's and Italy's in α_3 . It is particularly interesting to note that the relation in α_3 vector is dominated by the UK and, since we rejected the joint hypothesis of unit vectors in both Japan and the UK but accepted the hypothesis of unit vector in the UK, it is apparent that α_3 is the vector that contains that relation. The joint hypothesis of the US as a weakly exogenous variable and the unit vector in α_3 corresponding to the UK is accepted with $p = 0.78$ ($\chi^2(12) = 8.06$). This means that shocks to the US permanently influence the stochastic trends of the other 6 countries, but shocks to the UK have a purely transitory effect in the model. Additionally, the UK exclusively adjusts to the third cointegration relation.

The outcome of weak exogeneity of the US and the transitory effect of shocks to the UK can be also reached from an examination of t-values in the C matrix, which determines the significance of each country as a pushing force of the system. Table 11

presents the C matrix for Model 3(1) when no restrictions are posed on the α matrix. Following the bolded cells (the ones with significant t-values) in Table 11, we can see that shocks to the US are significant for all countries while shocks to the UK are insignificant for all countries and thus have a transitory effect.

The α, β', Π matrices and $\alpha_{\perp}, \tilde{\beta}_{\perp}, C$ matrices with restrictions according Model 3(1) and the US as weakly exogenous variable are presented in Tables 12 and 13 respectively. According to α_{\perp} in Table 14, the first three trends are driven by Canada, France, Germany, Italy and Japan. The force trend corresponds to the US. Figure 4 illustrates 4 common trends.

Based on the estimated C matrix the following conclusions are drawn: shocks to the US have a permanent influence on the other countries while the US itself is not influenced by shocks to those countries; the only outside shocks that influence the stochastic trends of Canada, France and the UK are shocks from the US; Germany and Japan transmit permanent shocks between each other and are influenced by shocks from France and the US; and finally, Canada, France, Germany and the US affect Italy's economy, while shocks to Italy do not affect any other country.

5. Conclusion

The study aims to identify the driving forces behind the international business cycle of G7 countries. It identifies the transmission of permanent shocks from one G7 country to another. The following objective is reached through an application of a cointegrated VAR model. The methodology identifies the pushing countries within the system, as well as the adjustment mechanism towards the steady-state. It allows the structuring of long-run information about the driving forces in the data via econometric identification restrictions on common long-run relations of the system.

The conclusion of the study is that the main pushing force in the system of international business cycles among G7 countries is the US, while the US itself is not influenced by any of the other countries. The secondary pushing forces are Germany and

France. At the same time, one of the main pulling forces towards the steady-state is a combined force of France, Germany and the US. The UK does not exhibit any permanent influence on any of the countries, and shocks by the UK have only a transitory effect on the countries in the system. Finally, the UK pulled back to the steady state by the third equilibrium mechanism between Germany, Italy, Japan, the UK and the US.

Table 1a: Specification Tests for the Unrestricted VAR(2) Model

Multivariate Tests:							
<u>Residual Autocorrelations:</u>							
LM ₁	$\chi^2(49) = 62.7373$	P-value = 0.090					
LM ₄	$\chi^2(49) = 39.8380$	P-value = 0.822					
<u>Normality:</u>							
LM	$\chi^2(14) = 218.9125$	P-value = 0.000					
<u>Trace Correlation</u> = 0.2236							
Univariate Tests:							
	Canada	France	Germany	Italy	Japan	UK	US
Mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0064	0.0044	0.0120	0.0063	0.0087	0.0085	0.0076
Skewness	-0.1879	-0.3737	4.0305	0.0434	-0.8421	0.4695	-0.1253
Kurtosis	3.2640	3.2659	35.5365	2.8684	6.0870	6.9547	4.3192
ARCH (1)	19.6840	0.7471	0.0477	3.3765	0.1475	8.2708	10.3046
Normality	1.6828	3.2857	137.3445	0.0582	21.8440	45.8578	11.1623
R-Squared	0.3722	0.3898	0.2013	0.4311	0.1830	0.2278	0.1575

Table 1b: Specification Tests for the Unrestricted VAR(2) Model after Correction for the Outliers

Multivariate Tests:							
<u>Residual Autocorrelations:</u>							
LM ₁	$\chi^2(49) = 63.7576$	P-value = 0.077					
LM ₄	$\chi^2(49) = 60.6248$	P-value = 0.123					
<u>Normality:</u>							
LM	$\chi^2(14) = 23.8204$	P-value = 0.048					
<u>Trace Correlation</u> = 0.4026							
Univariate Tests:							
	Canada	France	Germany	Italy	Japan	UK	US
Mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0061	0.0043	0.0068	0.0061	0.0075	0.0067	0.0068
Skewness	-0.1529	-0.4332	-0.1040	0.0217	-0.1435	-0.3906	-0.5004
Kurtosis	3.3680	3.5465	3.7977	2.9527	3.1057	3.8247	3.6335
ARCH (1)	16.1890	0.6464	3.8062	0.8443	1.8013	3.6331	13.7491
Normality	2.1020	4.6598	5.4381	0.1328	0.8739	5.7180	5.7648
R-Squared	0.4289	0.4224	0.7485	0.4627	0.3939	0.5119	0.3208

Table 2: SIC, HQ, AIC and Likelihood Ratio Tests for Lag Selection

Lags <i>k</i>	Log Likelihood	Information Criteria			LM(1) p-value	Normality LM (p-value)	Trace Correlation
		SIC	AIC	HQ			
1	-70.9766	-69.1531	-70.2285	-69.0434	0.002	0.000	0.3463
2	-71.7620	-68.1149	-70.2658	-67.8956	0.077	0.048	0.4026
3	-72.4393	-66.9687	-70.195	-66.6397	0.033	0.063	0.4605
4	-73.1263	-65.8321	-70.1339	-65.3935	0.493	0.009	0.5112

Note: Lag selection is performed after taking into account permanent dummies. SIC, HQ and AIC are Schwarz, Hannan-Quinn, and Akaike information criteria respectively. SIC, HQ and AIC are performed keeping effecting sample size constant.

Table 3: Trace Test Statistics for Determination of Cointegration Rank for the Unrestricted VAR(2) Model with Dummies

r	p - r	Eigen. Value	Trace	95% Critical Value	P-Value
0	7	0.3158	145.1760	125.4167	0.0015
1	6	0.2265	94.7088	95.5141	0.0570
2	5	0.1848	60.5575	69.6109	0.2192
3	4	0.1242	33.3861	47.7073	0.5400
4	3	0.0731	15.7527	29.8044	0.7348
5	2	0.0412	5.6519	15.4082	0.7376
6	1	0.0004	0.0556	3.8415	0.8136

Note: r is number of cointegrating relations and p-r is number of common stochastic trends in the system.

Table 4: Roots of the Companion Matrix

Modulus of Seven Largest Roots						
r=7	r=6	r=5	r=4	r=3	r=2	r=1
0.9975	1	1	1	1	1	1
0.9742	0.9737	1	1	1	1	1
0.9742	0.9737	0.9495	1	1	1	1
0.9100	0.9137	0.9495	0.9247	1	1	1
0.8703	0.8669	0.8781	0.9247	0.9273	1	1
0.8703	0.8669	0.8781	0.8974	0.9273	0.9251	1
0.5421	0.5417	0.5454	0.5400	0.5649	0.5740	0.5832

Table 5: Restrictions on Known β : Model 1 and Model 2

	Canada	France	Germany	Italy	Japan	UK	US	$\chi^2(\nu)$	p-value
Model 1									
$H_{1,1}$	0	0	1	1	1	1	1	5.74(2)	0.06
$H_{1,2}$	1	0	1	1	1	1	0	6.27(3)	0.10
$H_{1,3}$	1	1	1	1	1	1	0	0.96(1)	0.33
Model 2									
$H_{2,1}$	1	1	0	0	0	0	0	4.68(4)	0.32
$H_{2,2}$	0	1	0	0	0	1	0	8.05(4)	0.09
$H_{2,3}$	0	1	0	0	0	0	1	8.10(4)	0.09
$H_{2,4}$	0	0	1	0	0	1	0	5.59(4)	0.23
$H_{2,5}$	0	0	1	0	0	0	1	8.40(4)	0.08
$H_{2,6}$	0	0	0	0	1	1	0	8.35(4)	0.08
$H_{2,7}$	0	0	0	0	1	0	1	9.19(4)	0.06
$H_{2,8}$	0	0	0	0	0	1	1	8.32(4)	0.08
$H_{2,9}$	1	0	1	1	1	0	0	3.96(2)	0.14
$H_{2,10}$	1	0	0	0	0	1	1	6.71(3)	0.08
$H_{2,11}$	1	1	0	0	0	1	0	2.27(3)	0.52
$H_{2,12}$	1	1	0	0	0	1	0	1.82(3)	0.61
$H_{2,13}$	0	1	1	0	0	0	1	0.03(3)	0.99
$H_{2,14}$	1	1	1	0	0	0	1	0.02(2)	0.99
$H_{2,15}$	1	0	0	1	1	1	0	1.79(2)	0.41
$H_{2,16}$	0	1	1	1	0	1	0	1.94(2)	0.38
$H_{2,17}$	0	1	1	1	0	0	1	0.02(2)	0.99
$H_{2,18}$	1	1	1	1	0	0	1	0.02(1)	0.89
$H_{2,19}$	1	0	1	1	1	1	0	0.30(1)	0.58
$H_{2,20}$	0	0	1	1	1	1	1	2.33(1)	0.13
$H_{2,21}$	1	1	1	1	0	0	1	0.02(1)	0.90
$H_{2,22}$	0	1	1	1	1	0	1	0.003(1)	0.95

Table 6: Restriction on β' Matrix: Model 2 and Model 3

	Canada	France	Germany	Italy	Japan	UK	US
Model 2							
(1) $H_{2,13}$ and $H_{2,15}$, $\chi^2(5) = 2.11$, p-value = 0.83							
β_1	0	1	1	0	0	0	1
β_2	1	0	0	1	1	1	0
(2) $H_{2,14}$ and $H_{2,15}$, $\chi^2(4) = 1.98$, p-value = 0.74							
β_1	1	1	1	0	0	0	1
β_2	1	0	0	1	1	1	0
(3) $H_{2,17}$ and $H_{2,15}$, $\chi^2(4) = 2.11$, p-value = 0.72							
β_1	0	1	1	1	0	0	1
β_2	1	0	0	1	1	1	0
(4) $H_{1,3}$ and $H_{2,13}$, $\chi^2(3) = 0.02$, p-value = 0.99							
β_1	1	1	1	1	1	1	0
β_2	0	1	1	0	0	0	1
Model 3							
(1) $H_{2,13}$, $H_{1,2}$ and $H_{1,1}$ ($H_{2,20}$), $\chi^2(2) = 0.01$, p-value = 0.99							
β_1	0	1	1	0	0	0	1
β_2	1	0	1	1	1	1	0
β_3	0	0	1	1	1	1	1
(2) $H_{2,14}$, $H_{2,22}$, $\chi^2(2) = 0.003$, p-value = 0.99							
β_1	1	1	1	0	0	0	1
β_2	0	1	1	1	1	0	1
β_3	0	1	0	1	0	1	1
(3) $H_{2,14}$, $H_{2,22}$, $\chi^2(1) = 0.002$, p-value = 0.96							
β_1	1	1	1	0	0	0	1
β_2	0	1	1	1	1	0	1
β_3	0	0	1	1	1	1	1

Table 7: Zero Row Restrictions on α Matrix

	Canada	France	Germany	Italy	Japan	UK	US	$\chi^2(\nu)$	p-value
Model 1									
H_1^α	1	0	0	0	0	0	0	0.31(1)	0.58
H_2^α	0	0	0	0	0	0	1	0.02(1)	0.88
H_3^α	1	0	0	0	0	0	1	0.32(2)	0.85
Model 2									
H_4^α	1	0	0	0	0	0	0	3.77(2)	0.15
H_5^α	0	0	0	0	0	1	0	5.65(2)	0.06
H_6^α	0	0	0	0	0	0	1	0.28(2)	0.87
H_7^α	1	0	0	0	0	0	1	5.00(4)	0.29
H_8^α	1	0	0	0	0	1	0	12.29(4)	0.02
H_9^α	0	0	0	0	0	1	1	6.27 (4)	0.18
H_{10}^α	1	0	0	0	0	1	1	14.60 (6)	0.02
Model 3									
H_{11}^α	1	0	0	0	0	0	0	6.59(3)	0.09
H_{12}^α	0	0	0	0	0	0	1	0.30(3)	0.96
H_{13}^α	1	0	0	0	0	0	1	9.42(6)	0.15

Table 8: Unit Vector Restrictions on α Matrix

	Canada	France	Germany	Italy	Japan	UK	US	$\chi^2(\nu)$	p-value
Model 3									
H_{14}^α	1	0	0	0	0	0	0	10.81(4)	0.03
H_{15}^α	0	1	0	0	0	0	0	15.66(4)	0.00
H_{16}^α	0	0	1	0	0	0	0	10.28(4)	0.04
H_{17}^α	0	0	0	1	0	0	0	11.54(4)	0.02
H_{18}^α	0	0	0	0	1	0	0	9.36(4)	0.05
H_{19}^α	0	0	0	0	0	1	0	6.27(4)	0.18
H_{20}^α	0	0	0	0	0	0	1	21.90(4)	0.00
H_{21}^α	0	0	0	0	1	1	0	18.58(8)	0.02

Table 9: Trace Test Statistics for Determination of Cointegration Rank for the Unrestricted VAR(2) Model with Dummies and the US as Weakly Exogenous Variable

r	p - r	Eigen. Value	Trace	95% Critical Value	P-Value
0	6	0.3156	143.3927	109.0436	0.0000
1	5	0.2250	92.9482	81.1561	0.0045
2	4	0.1847	59.0559	57.2729	0.0342
3	3	0.1227	31.9013	37.3965	0.1740
4	2	0.0730	14.4922	21.5300	0.3394
5	1	0.0327	4.4174	9.3404	0.3782

Note: r is number of cointegrating relations and p-r is number of common stochastic trends in the system. Critical values are generated by Harboe, Johansen and Rahbek (1997).

Table 10: Unrestricted α Matrix for Model 3(1)

	α_1	α_2	α_3
Canada	-0.0684 (-2.9611)	0.1129 (2.3009)	0.0176 (0.8159)
France	-0.0399 (-2.4598)	-0.0684 (-1.9852)	0.0237 (1.5632)
Germany	0.0019 (0.0741)	-0.1618 (-2.9393)	0.0090 (0.3740)
Italy	0.0132 (0.5765)	-0.2669 (-5.4882)	0.0168 (0.7851)
Japan	-0.0780 (-2.8169)	-0.0142 (-0.2419)	-0.0507 (-1.9638)
UK	-0.0554 (-2.2231)	-0.1053 (-1.9884)	0.1155 (4.9645)
US	0.0105 (0.4141)	-0.0063 (-0.1179)	0.0035 (0.1475)

* t-values are in the brackets

Table 11: C Matrix for Model 3(1) with Unrestricted α Coefficients

	$\sum \varepsilon_{1t}$	$\sum \varepsilon_{2t}$	$\sum \varepsilon_{3t}$	$\sum \varepsilon_{4t}$	$\sum \varepsilon_{5t}$	$\sum \varepsilon_{6t}$	$\sum \varepsilon_{7t}$
Canada	1.2009 (3.8336)	-0.7038 (-1.0002)	0.0083 (0.0257)	0.7579 (1.5235)	-0.1552 (-0.6351)	-0.2667 (-0.4799)	1.6331 (2.2013)
France	0.1457 (0.9347)	0.0654 (0.1868)	-0.2059 (-1.2854)	0.1614 (0.6523)	0.0518 (0.4260)	-0.0543 (-0.1964)	1.1315 (3.0653)
Germany	0.0636 (0.3766)	-1.0263 (-2.7052)	0.4768 (2.7468)	-0.1606 (-0.5987)	0.3442 (2.6130)	0.3126 (1.0432)	0.8350 (2.0875)
Italy	0.5026 (2.3144)	-1.1565 (-2.3705)	-0.4705 (-2.1078)	0.6792 (1.9694)	0.2761 (1.6297)	0.1855 (0.4814)	1.1405 (2.2173)
Japan	-0.2102 (-0.7877)	-1.8501 (-3.0860)	-0.6502 (-2.3703)	0.4176 (0.9854)	0.8335 (4.0043)	0.7302 (1.5420)	1.2378 (1.9584)
UK	0.0182 (0.1063)	0.0092 (0.0240)	0.2288 (1.2989)	-0.1723 (-0.6331)	0.1057 (0.7905)	0.0131 (0.0432)	1.1843 (2.9171)
US	0.1820 (0.8391)	-0.3086 (-0.6334)	-0.0476 (-0.2134)	0.1162 (0.3373)	0.1837 (1.0858)	0.0566 (0.1471)	1.5398 (2.9974)

* t-values are in the brackets

Table 12: α, β', Π Matrices: Model 3(1), Zero Row Restriction on α

Π Matrix							
	Canada	France	Germany	Italy	Japan	UK	US
Canada	-0.0602 (-2.3516)	-0.0735 (-3.1694)	-0.0001 (-0.0161)	0.1098 (2.1720)	-0.0618 (-2.3982)	0.0049 (0.1947)	0.0825 (2.3797)
France	0.0359 (1.9932)	-0.0418 (-2.5596)	-0.0224 (-4.0296)	-0.0764 (-2.1480)	0.0351 (1.9353)	-0.0356 (-2.0205)	0.0605 (2.4803)
Germany	0.0842 (2.9229)	-0.0004 (-0.0147)	-0.0287 (-3.2197)	-0.1635 (-2.8735)	0.0848 (2.9256)	-0.0364 (-1.2927)	0.0074 (0.1894)
Italy	0.1402 (5.5014)	0.0130 (0.5616)	-0.0425 (-5.3975)	-0.2736 (-5.4364)	0.1409 (5.4978)	-0.0648 (-2.6018)	0.0039 (0.1144)
Japan	0.0067 (0.2182)	-0.0805 (-2.8921)	-0.0384 (-4.0449)	0.0054 (0.0884)	0.0097 (0.3144)	0.0521 (1.7343)	0.0209 (0.5026)
UK	0.0557 (2.0058)	-0.0586 (-2.3295)	-0.0184 (-2.1501)	-0.1458 (-2.6610)	0.0499 (1.7894)	-0.1371 (-5.0510)	0.1672 (4.4503)
US	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
β' Matrix							
	Canada	France	Germany	Italy	Japan	UK	US
β_1	0.0000 (0.0000)	1.0000 (0.0000)	0.3294 (3.3819)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	-0.9129 (-11.1055)
β_2	-0.5230 (-9.7934)	0.0000 (0.0000)	0.1850 (4.1430)	1.0000 (0.0000)	-0.5290 (-14.3803)	0.1809 (2.7566)	0.0000 (0.0000)
β_3	0.0000 (0.0000)	0.0000 (0.0000)	-0.1745 (-1.3221)	0.3342 (1.3565)	0.0539 (0.3193)	1.0000 (0.0000)	-0.9657 (-6.9841)
α Matrix							
		α_1	α_2	α_3			
	Canada	-0.0735 (-3.1694)	0.1151 (2.3516)	-0.0160 (-0.7191)			
	France	-0.0418 (-2.5596)	-0.0687 (-1.9932)	-0.0232 (-1.4844)			
	Germany	-0.0004 (-0.0147)	-0.1611 (-2.9229)	-0.0073 (-0.2919)			
	Italy	0.0130 (0.5616)	-0.2681 (-5.5014)	-0.0163 (-0.7399)			
	Japan	-0.0805 (-2.8921)	-0.0128 (-0.2182)	0.0545 (2.0442)			
	UK	-0.0586 (-2.3295)	-0.1064 (-2.0058)	-0.1178 (-4.8993)			
	US	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)			

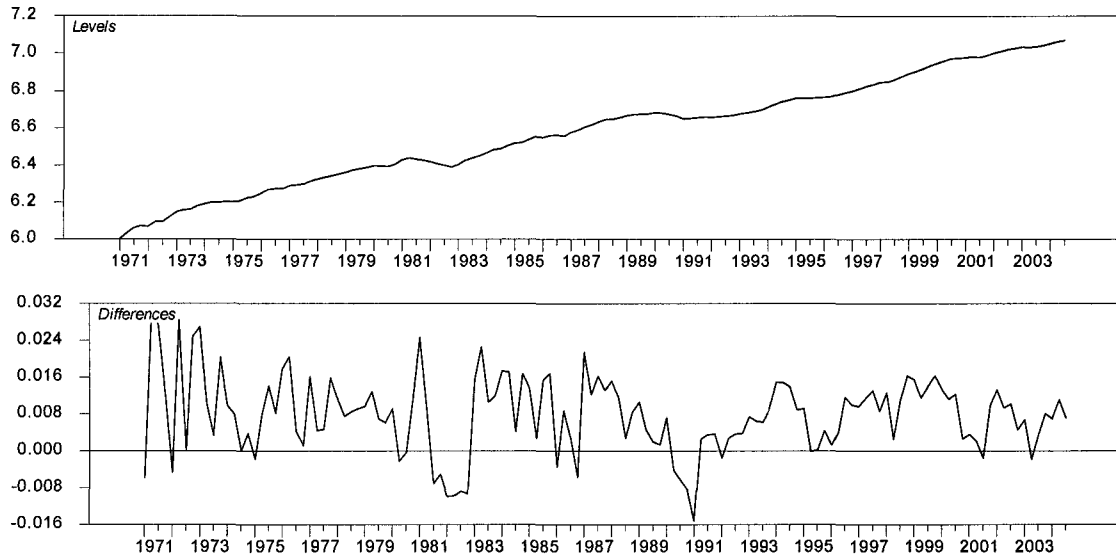
Table 13: $\alpha_{\perp}, \tilde{\beta}_{\perp}, C$ Matrices: Model 3(1), Zero Row Restriction on α

C							
	$\sum \varepsilon_{1t}$	$\sum \varepsilon_{2t}$	$\sum \varepsilon_{3t}$	$\sum \varepsilon_{4t}$	$\sum \varepsilon_{5t}$	$\sum \varepsilon_{6t}$	$\sum \varepsilon_{7t}$
Canada	1.1190 (3.3558)	-0.9859 (-1.2396)	-0.0719 (-0.2130)	0.8600 (1.5766)	-0.2381 (-0.8869)	-0.1825 (-0.3129)	1.8376 (2.1518)
France	0.0961 (0.5729)	-0.1025 (-0.2562)	-0.2488 (-1.4648)	0.2188 (0.7973)	0.0057 (0.0419)	-0.0052 (-0.0177)	1.2513 (2.9122)
Germany	0.0157 (0.0883)	-1.1692 (-2.7539)	0.4419 (2.4515)	-0.1169 (-0.4015)	0.3095 (2.1596)	0.3599 (1.1560)	0.9319 (2.0441)
Italy	0.4457 (1.9099)	-1.3332 (-2.3952)	-0.5124 (-2.1682)	0.7347 (1.9246)	0.2319 (1.2338)	0.2389 (0.5854)	1.2642 (2.1152)
Japan	-0.2667 (-0.9464)	-2.0110 (-2.9921)	-0.6828 (-2.3927)	0.4635 (1.0055)	0.7981 (3.5174)	0.7787 (1.5801)	1.3489 (1.8690)
UK	-0.0247 (-0.1361)	-0.1659 (-0.3839)	0.1758 (0.9583)	-0.1002 (-0.3379)	0.0473 (0.3244)	0.0609 (0.1921)	1.3159 (2.8354)
US	0.1110 (0.4745)	-0.5342 (-0.9578)	-0.1131 (-0.4778)	0.1975 (0.5163)	0.1179 (0.6261)	0.1242 (0.3036)	1.7070 (2.8503)
$\tilde{\beta}_{\perp}'$							
	Canada	France	Germany	Italy	Japan	UK	US
$\tilde{\beta}_{\perp 1}$	0.9555	0.2769	0.7451	1.5973	2.3430	0.0225	0.5722
$\tilde{\beta}_{\perp 2}$	-1.2504	-0.5772	-0.0758	-2.1323	-2.7490	0.2104	-0.6597
$\tilde{\beta}_{\perp 3}$	1.1883	0.2923	-0.6673	0.5961	-0.3565	-0.2196	0.0794
$\tilde{\beta}_{\perp 4}$	1.8376	1.2513	0.9319	1.2642	1.3489	1.3159	1.7070
α_{\perp}'							
$\alpha_{\perp 1}$	1.0000	-1.8319	0.9202	0.2973	-0.0082	0.1231	0.0000
$\alpha_{\perp 2}$	0.8196	-0.8036	1.0000	0.0193	-0.2395	-0.1281	0.0000
$\alpha_{\perp 3}$	1.0000	-0.2021	0.2518	0.5050	-0.4458	-0.3874	0.0000
$\alpha_{\perp 4}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

* t-values are in the brackets

Figure 1a: Log Real GDP of Canada, 1st Differences of the Series and Residuals

LYCN



DLYCN

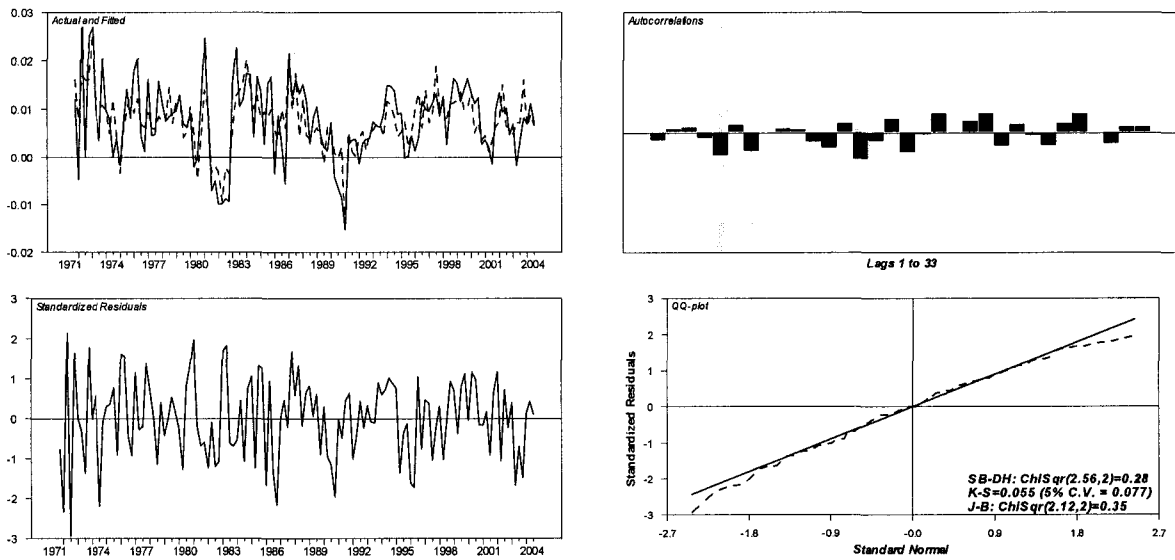
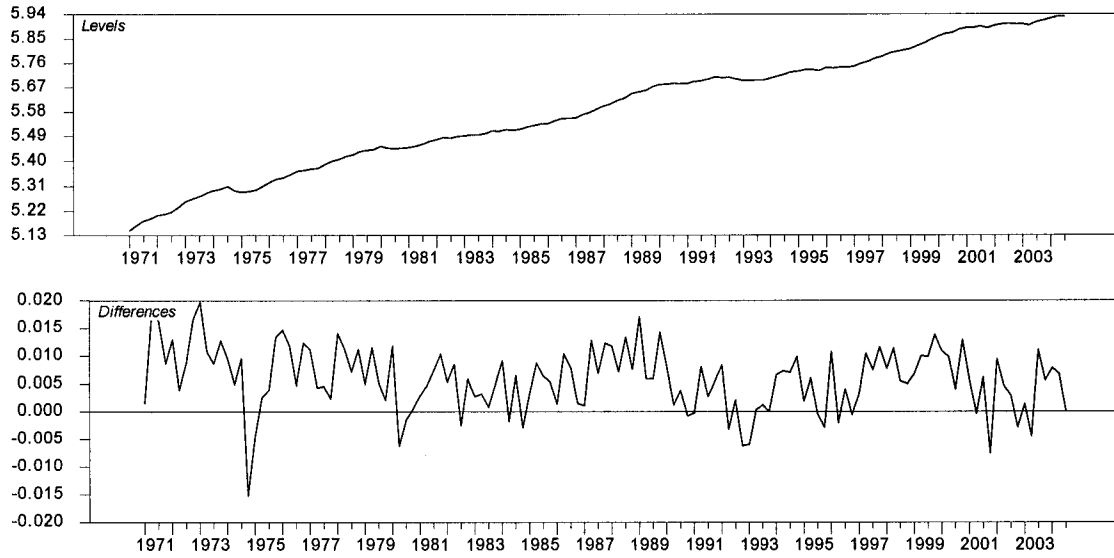


Figure 1b: Log Real GDP of France, 1st Differences of the Series and Residuals

LYFR



DLYFR

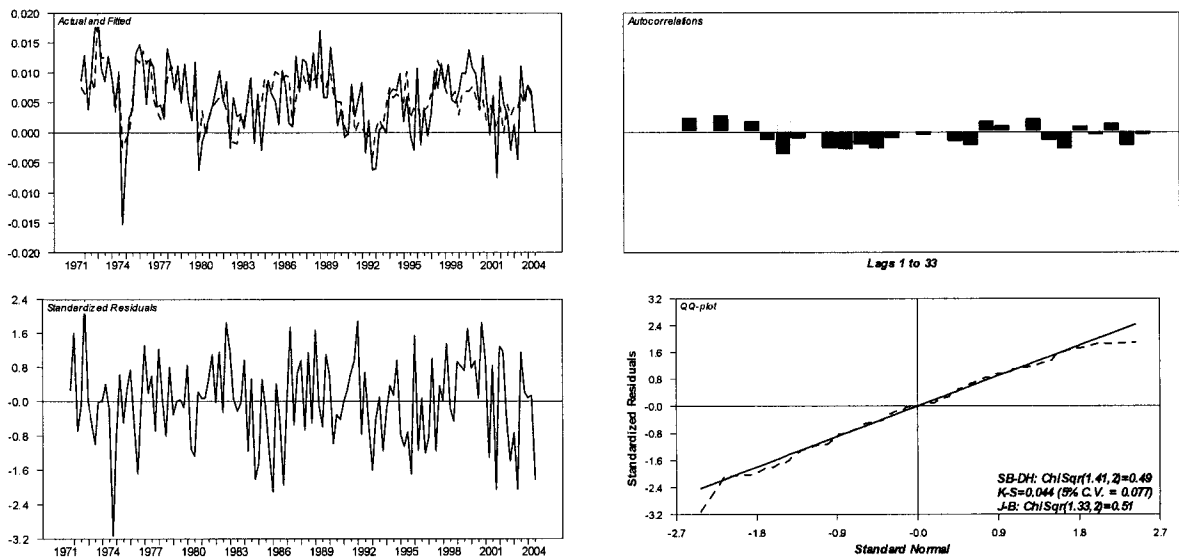
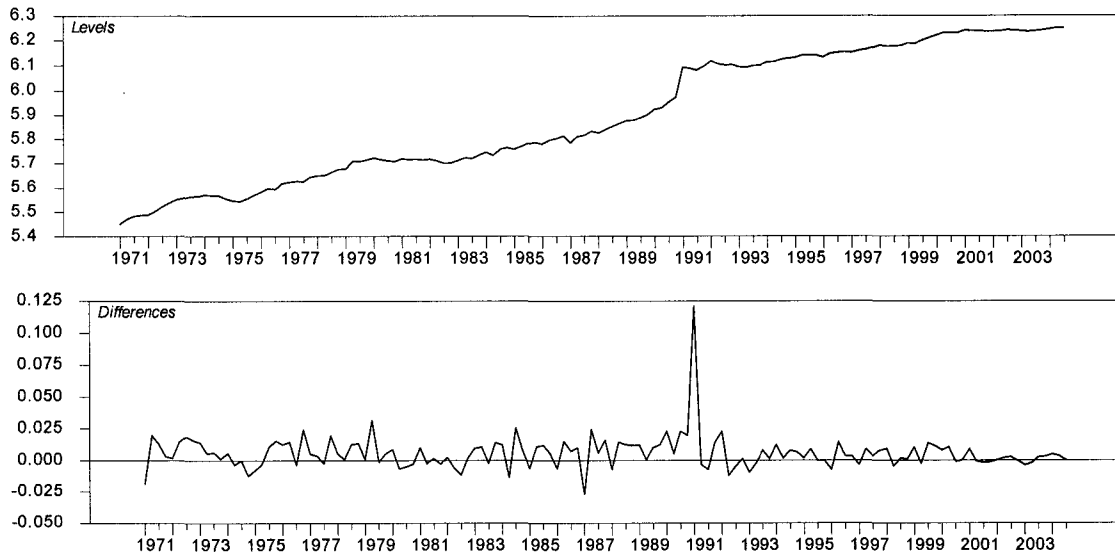


Figure 1c: Log Real GDP of Germany, 1st Differences of the Series and Residuals

LYGM



DLYGM

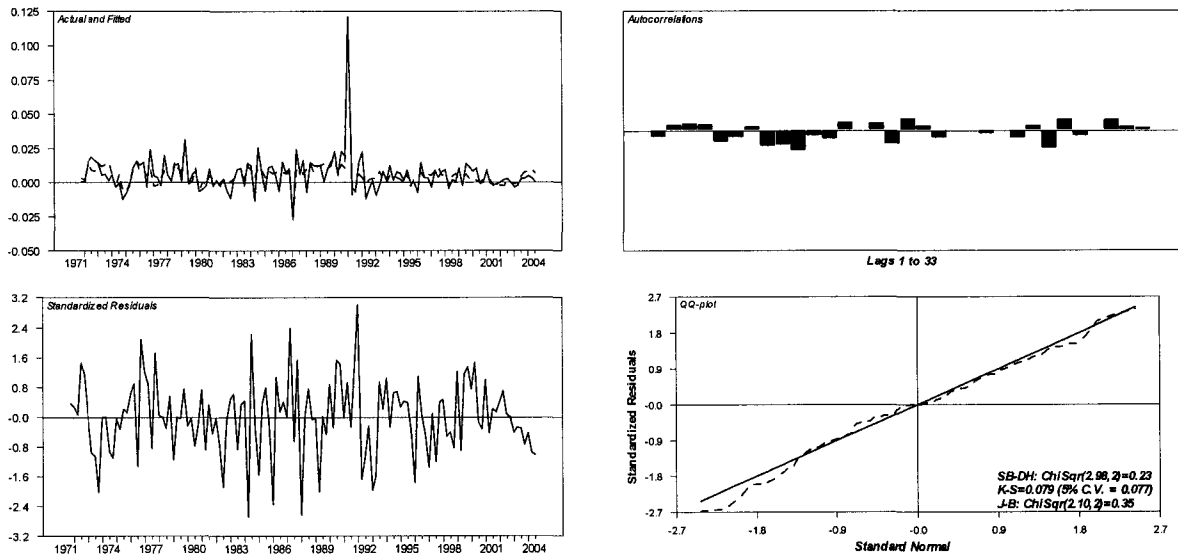
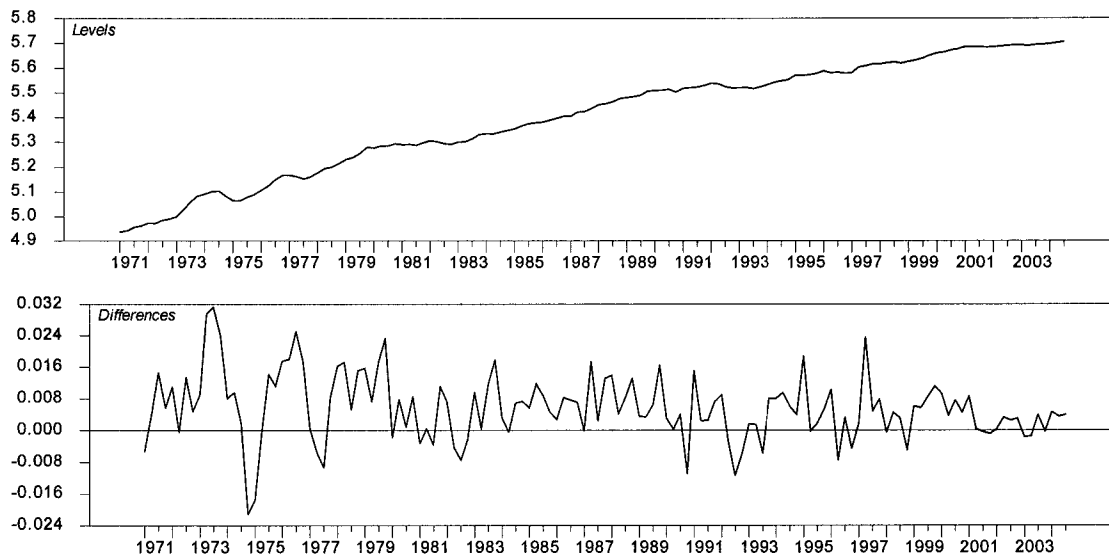


Figure 1d: Log Real GDP of Italy, 1st Differences of the Series and Residuals

LYIT



DLYIT

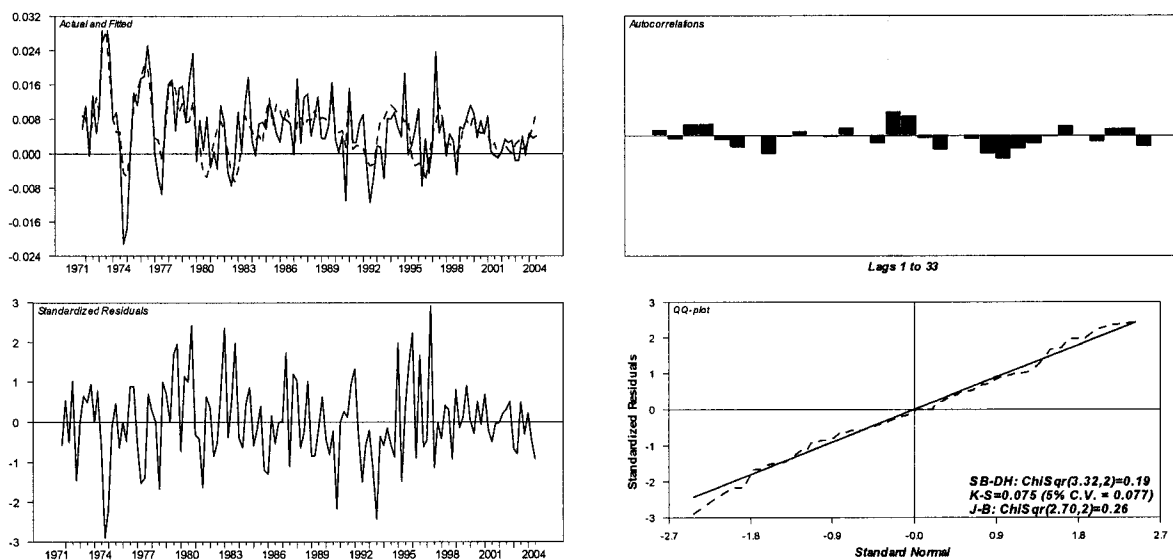
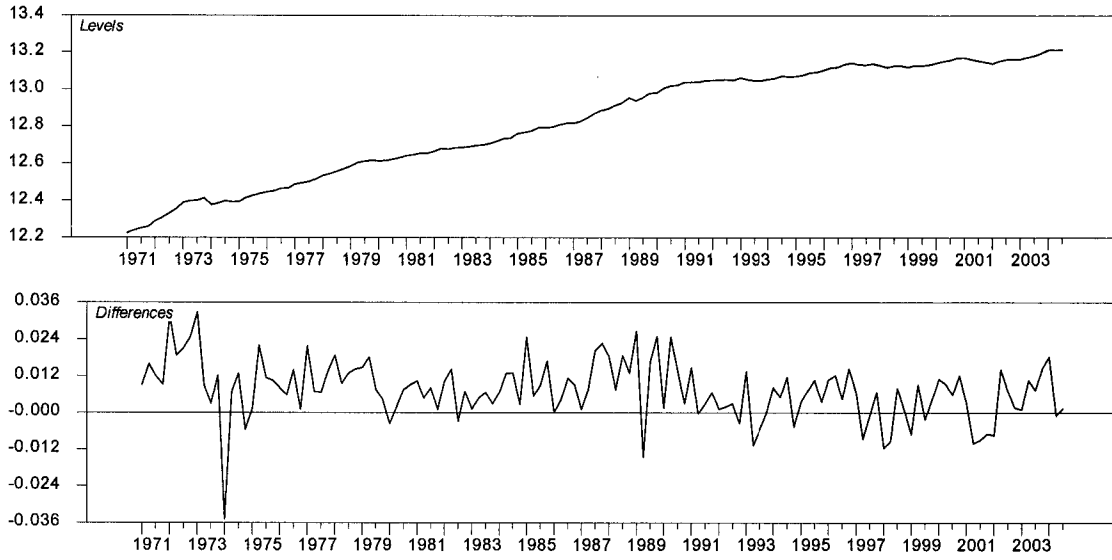


Figure 1e: Log Real GDP of Japan, 1st Difference of the Series and Residuals

LYJP



DLYJP

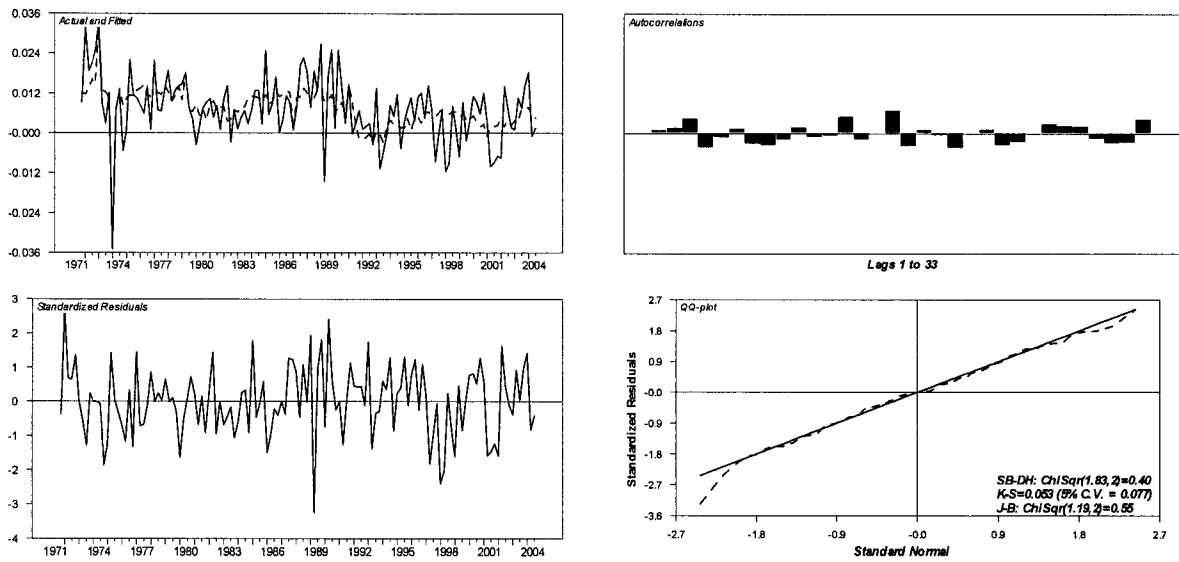
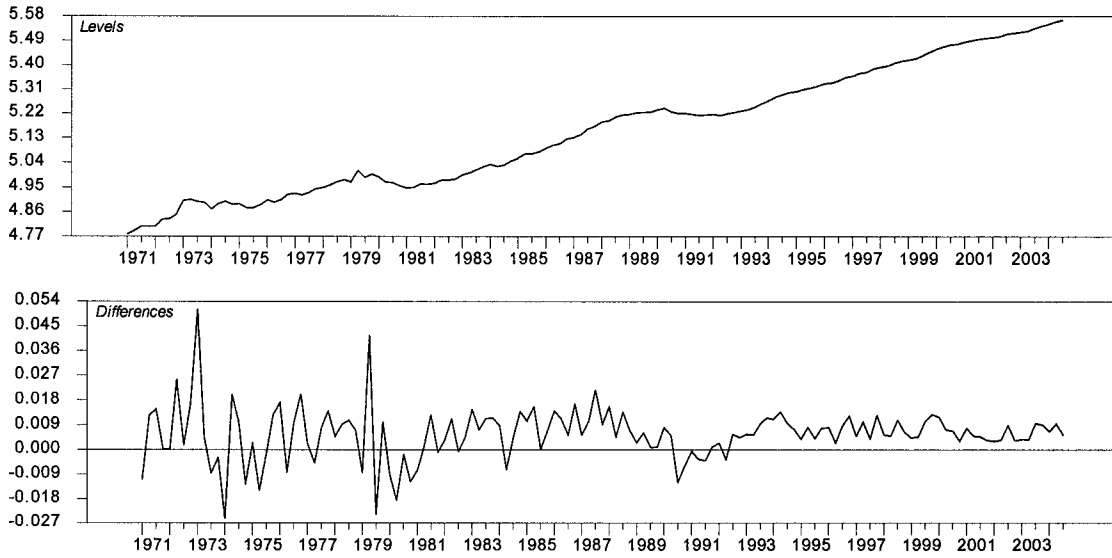


Figure 1f: Log Real GDP of the UK, 1st Difference of the Series and Residuals

LYUK



DLYUK

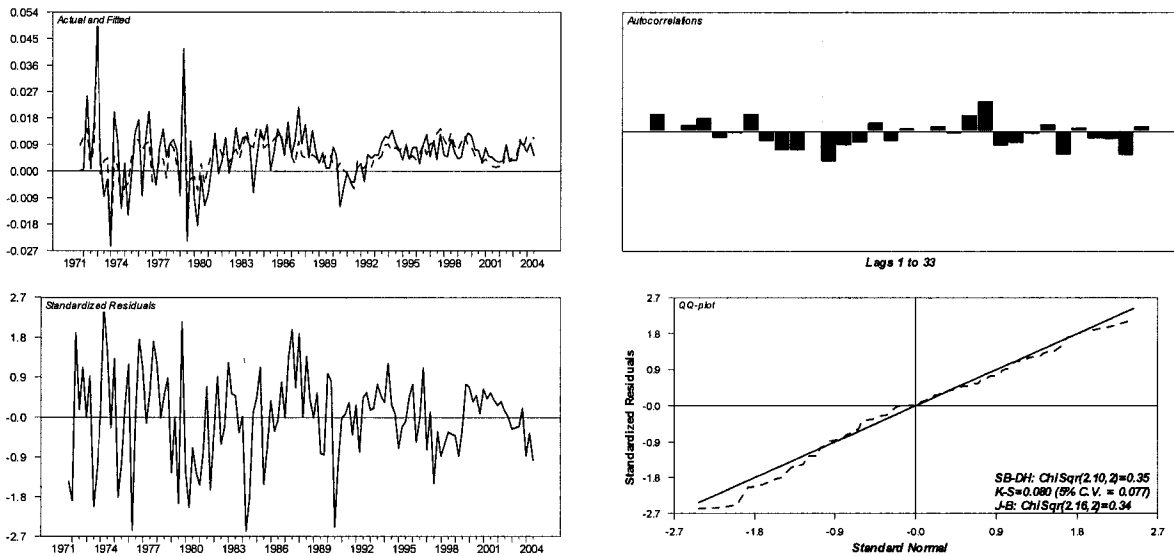
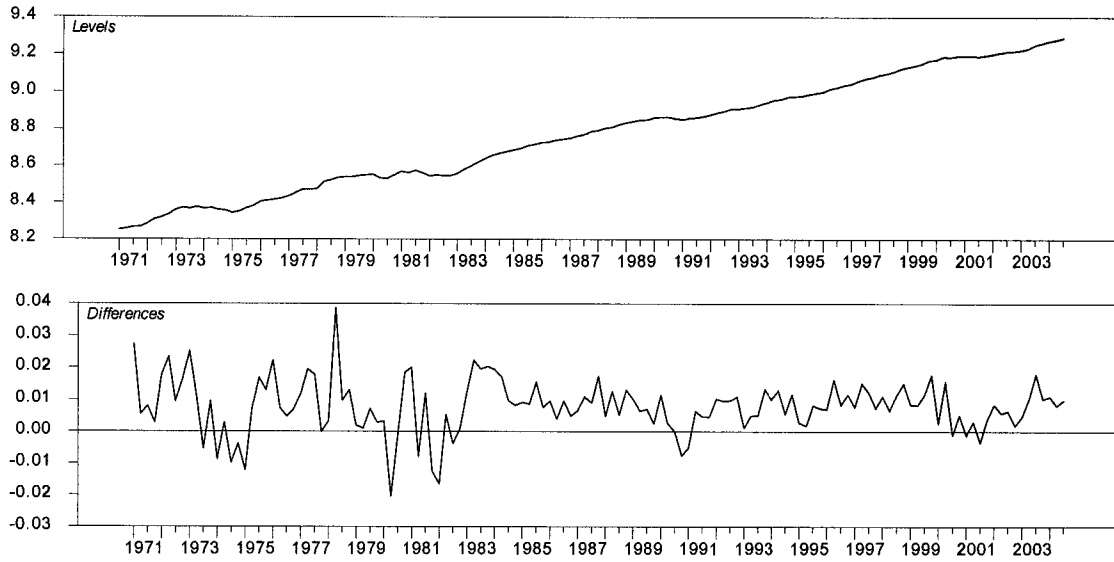


Figure 1g: Log Real GDP of the US, 1st Difference of the Series and Residuals

LYUS



DLYUS

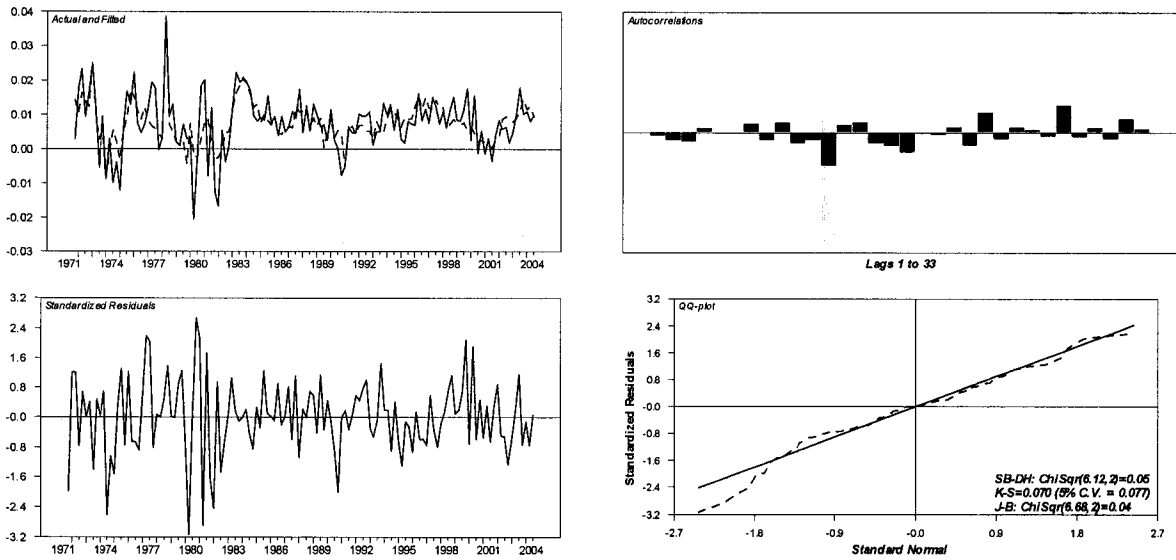


Figure 2: Pushing and Pulling Forces within Cointegrated VAR Model for Two Dimensional $x_{it} = (x_{1t}, x_{2t})$ System

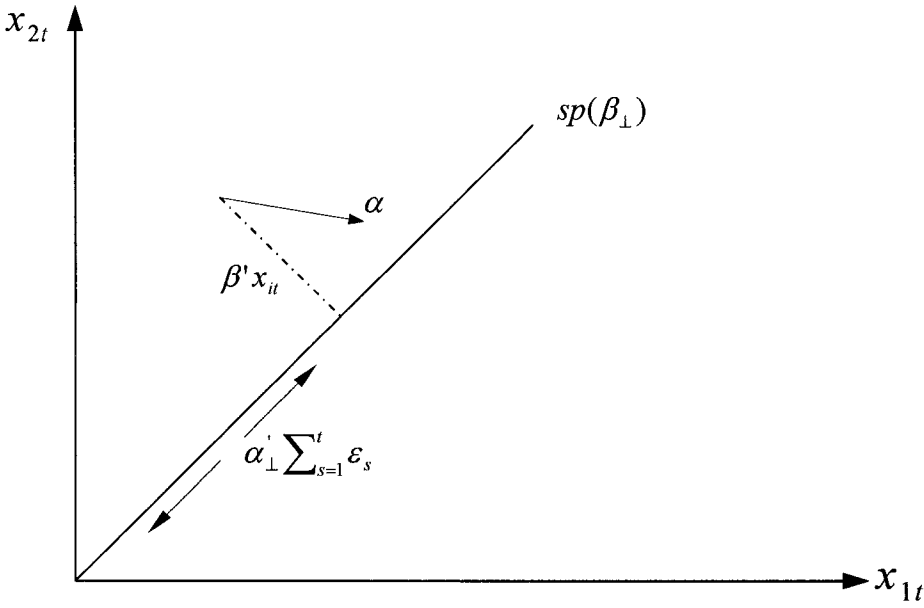


Figure 3: Recursively Estimated Trace Statistics

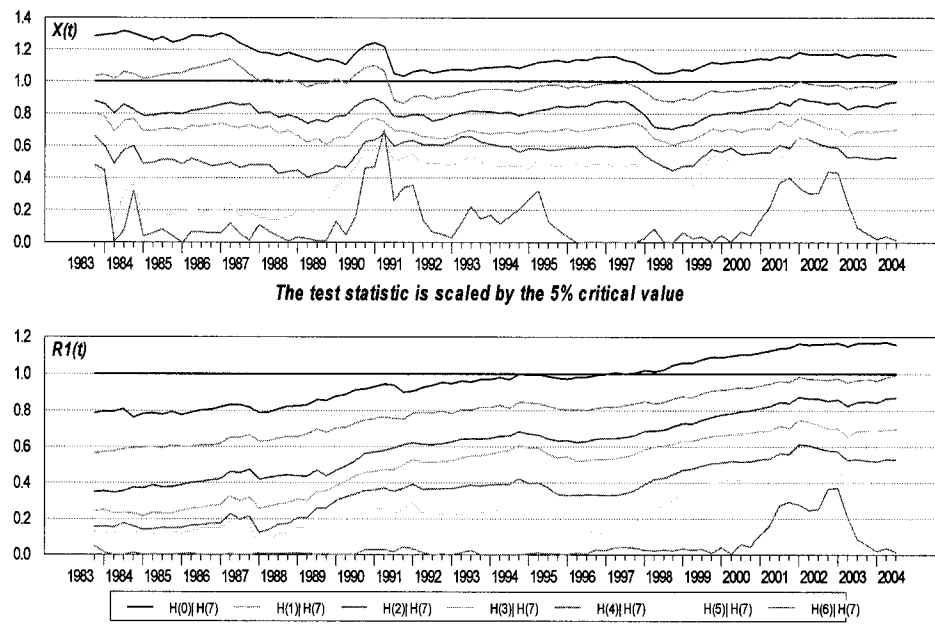
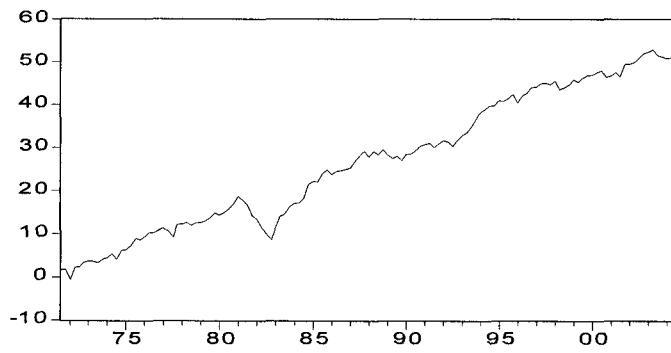
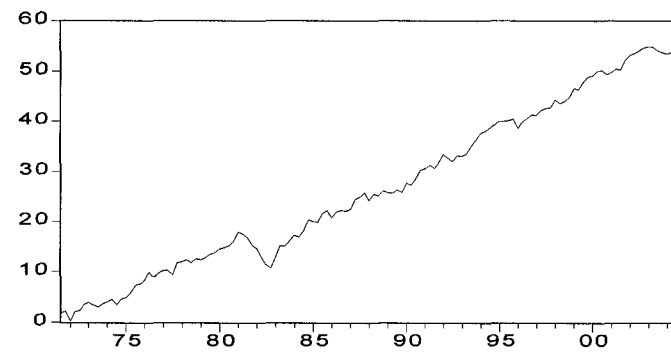


Figure 4: Four Common Stochastic Trends

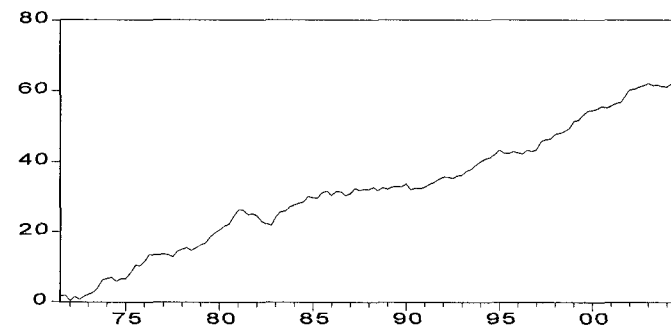
1st Common Trend



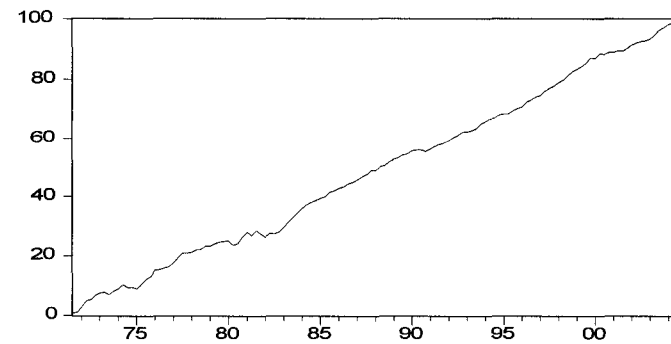
2nd Common Trend



3rd Common Trend



4th Common Trend



Appendix

A1 Representation and Estimation

To estimate the parameters, as well as unobserved components of the model the state-space representation of the model is used to apply Kalman filtering and Kim's (1993, 1994) approximate maximum likelihood estimation algorithm. The model is estimated in differences and is written in deviations from means:

$$\Delta y_{it} = \Delta Y_{it} - \Delta \bar{Y}_i.$$

The measurement equation of multivariate unobserved component model is:

$$\Delta y_t = H\beta_t. \tag{1A}$$

The transition equation of the model is:

$$\beta_t = \tilde{\mu} + F\beta_{t-1} + V_t, \tag{2A}$$

and

$$E(V_t V_t') = Q. \tag{3A}$$

Where:

$$\Delta y_t = \begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \\ \Delta y_{4t} \\ \Delta y_{5t} \\ \Delta y_{6t} \\ \Delta y_{7t} \end{bmatrix}, \quad H = \begin{bmatrix} \gamma_1 & a_1 & -a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & a_2 & -a_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_3 & a_3 & -a_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma_4 & a_4 & -a_4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_5 & a_5 & -a_5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \gamma_6 & a_6 & -a_6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \gamma_7 & a_7 & -a_7 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\beta_t = \begin{bmatrix} \Delta\tau_t^w \\ c_t^w \\ c_{t-1}^w \\ z_{1t} \\ z_{2t} \\ z_{3t} \\ z_{4t} \\ z_{5t} \\ z_{6t} \\ z_{7t} \end{bmatrix}, \quad \tilde{\mu} = \begin{bmatrix} \mu_{S_{1t}} \\ \lambda S_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} \phi^\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi^c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_7 \end{bmatrix},$$

$$V_t = \begin{bmatrix} v_t \\ u_t \\ 0 \\ \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \\ \eta_{5t} \\ \eta_{6t} \\ \eta_{7t} \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{u,S_{2t}}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta_1}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\eta_2}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_3}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_4}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_5}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_6}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_7}^2 \end{bmatrix}$$

where $z_{it} = \Delta\tau_{it} + \Delta c_{it}$ with $v_{it} \sim iidN(0, \sigma_{v_{it}}^2)$ residuals. $\xi_t \sim iidN(0,1)$ is the residual term of $\Delta\tau_t^w$. $\Delta\tau_t^w, c_t^w, z_{it}$ are assumed to follow first order autoregressive process with $\phi^\tau, \phi^c, \phi_i$ autoregressive parameters.

The model involve two unobserved Markov-switching variables S_{1t} and S_{2t} . However it can be represented with single Markov-switching state variable defined such that:

$$S_t = 1 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 0$$

$$S_t = 2 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 1$$

$$S_t = 3 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 0$$

$$S_t = 4 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 1$$

In this case parameters of the matrices $\tilde{\mu}$ and Q depend on the four-state Markov-switching variable S_t with transition probabilities given in by following matrix p :

$$p = \begin{bmatrix} p_{11} & p_{21} & p_{31} & p_{41} \\ p_{12} & p_{22} & p_{32} & p_{42} \\ p_{13} & p_{23} & p_{33} & p_{43} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{bmatrix}$$

where $p_{ij} = \Pr[S_t = j | S_{t-1} = i]$ and $\sum_{j=1}^4 p_{ij} = 1$. Transition probabilities p_{ij} are functions of q_1, p_1, q_2 and p_2 . Assuming independence between S_{1t} and S_{2t} ,

$$p_{11} = \Pr[S_t = 1 | S_{t-1} = 1] = \Pr[S_{1t} = 0 | S_{1,t-1} = 0] \Pr[S_{2t} = 0 | S_{2,t-1} = 0] = q_1 q_2$$

The (1A)-(3A) state-space model is estimated with Kim (1994) approximate maximum likelihood estimation algorithm. If the state variable S_t would be observed the state-space model would be linear and Gaussian, and calculation of the exact likelihood function with the Kalman filter would be possible. However in the stat-space model with Markov-switching the forecast is formed not only based on the previous information, as it is done in Kalman filter, but also based on the random, unobserved variable S_t taking on the value j and S_{t-1} taking on the value i . In this case the model becomes nonlinear and the calculation of the exact likelihood function using Kalman filter is computationally infeasible. For more detailed discussion on the nature of approximation and the Bayesian alternative to the estimation procedure readers are referred to Kim and Nelson (1999b).

The Kalman filter equations together with Kim (1994) method to approximate the likelihood function conditional on $S_t = j$ and $S_{t-1} = i$ are given by:

$$\beta_{t|t-1}^{(i,j)} = \tilde{\mu}_{S_t} + F\beta_{t-1|t-1}^i \quad (4A)$$

$$P_{t|t-1}^{(i,j)} = FP_{t-1|t-1}^i F' + Q^j \quad (5A)$$

$$\eta_{t|t-1}^{(i,j)} = y_t - H\beta_{t|t-1}^{(i,j)} \quad (6A)$$

$$f_{t|t-1}^{(i,j)} = HP_{t|t-1}^{(i,j)} H' \quad (7A)$$

$$\beta_{t|t}^{(i,j)} = \beta_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} H' [f_{t|t-1}^{(i,j)} J^{-1}]^{-1} \eta_{t|t-1}^{(i,j)} \quad (8A)$$

$$P_{t|t}^{(i,j)} = (I - P_{t|t-1}^{(i,j)} H' [f_{t|t-1}^{(i,j)} J^{-1}]) HP_{t|t-1}^{(i,j)} \quad (9A)$$

were $\beta_{t|t-1}^{(i,j)}$ is an inference on β_t based on information up to time $(t-1)$ and $\beta_{t|t}^{(i,j)}$ is an inference on β_t based on information up to time t , with corresponding $P_{t|t-1}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ mean square error matrices, conditional on $S_t = j$ and $S_{t-1} = i$ respectively. $\eta_{t|t-1}^{(i,j)}$ is the conditional error of y_t based on information $(t-1)$ and $f_{t|t-1}^{(i,j)}$ is the conditional variance of $\eta_{t|t-1}^{(i,j)}$.

For each date t the proposed algorithm calculates a battery of 4^2 forecasts, corresponding to every possible value for i and j . Thus each iteration of the Kalman filter produces a 4-fold increase in the number of cases to consider. To keep Kalman filter operable, at the end of each time period, we need to collapse 4^2 posteriors $\beta_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ into 4 posteriors $\beta_{t|t}^j$ and $P_{t|t}^j$. Collapsing requires the following approximation:

$$\beta_{t|t}^j = \frac{\sum_{i=1}^4 \Pr[S_{t-1} = i, S_t = j | \Omega_t] \beta_{t|t}^{(i,j)}}{\Pr[S_t = 1 | \Omega_t]},$$

and

$$P_{t|t}^j = \frac{\sum_{i=1}^4 \Pr[S_{t-1} = i, S_t = j | \Omega_t] \{P_{t|t}^{(i,j)} + (\beta_{t|t}^j - \beta_{t|t}^{(i,j)})(\beta_{t|t}^j - \beta_{t|t}^{(i,j)})'\}}{\Pr[S_t = 1 | \Omega_t]},$$

where Ω_t refers to information available at time t .

To obtain the probability terms that are necessary to construct the approximations Hamilton (1989) filter is employed as a three-step procedure. To initialize the filter the steady-state probabilities π_j are used, such as:

$$\pi_j = \Pr[S_0 = j | \Omega_0] = \frac{1 - p_{jj}}{1 - p_{ii} - p_{jj}}$$

Step 1:

At the beginning of the i^{th} iteration, given $\Pr[S_{t-1} = i | \Omega_{t-1}]$, calculate

$$\Pr[S_t = j, S_{t-1} = i | \Omega_{t-1}] = \Pr[S_t = j | S_{t-1} = i] \Pr[S_{t-1} = i | \Omega_{t-1}]$$

Step 2:

Consider the joint density of Δy_t , S_t and S_{t-1} such as:

$$f(\Delta y_t, S_t = j, S_{t-1} = i | \Omega_{t-1}) = f(\Delta y_t | S_t = j, S_{t-1} = i, \Omega_{t-1}) \Pr[S_t = j, S_{t-1} = i | \Omega_{t-1}]$$

The marginal density of Δy_t is obtained from the joint density by:

$$\begin{aligned} f(\Delta y_t | \Omega_{t-1}) &= \sum_{i=1}^4 \sum_{j=1}^4 f(\Delta y_t, S_t = j, S_{t-1} = i | \Omega_{t-1}) \\ &= \sum_{i=1}^4 \sum_{j=1}^4 f(\Delta y_t | S_t = j, S_{t-1} = i, \Omega_{t-1}) \Pr[S_t = j, S_{t-1} = i | \Omega_{t-1}] \end{aligned}$$

where the conditional density $f(\Delta y_t | S_t = j, S_{t-1} = i, \Omega_{t-1})$ is obtained through the prediction-error decomposition:

$$f(\Delta y_t | S_t = j, S_{t-1} = i, \Omega_{t-1}) = (2\pi)^{-T/2} |f_{t|t-1}^{(i,j)}|^{-1/2} \exp\left\{-\frac{1}{2} \eta_{t|t-1}^{(i,j)'} f_{t|t-1}^{(i,j)-1} \eta_{t|t-1}^{(i,j)}\right\}.$$

where $\eta_{t|t-1}^{(i,j)}$ and $f_{t|t-1}^{(i,j)}$ are defined by Kalman equations (6A) and (7A).

As a result of the Step 2 we can obtain the log likelihood function, which can be maximized with respect to the parameters of the model. The log likelihood function is defined as:

$$\ln L = \sum_{t=1}^T \ln(f(\Delta y_t | \Omega_{t-1})).$$

Step 3:

Once Δy_t is observed at the end of time t the filter updates the probability terms:

$$\begin{aligned}
\Pr[S_t = j, S_{t-1} = i | \Omega_{t-1}] &= \Pr[S_t = j | S_{t-1} = i | \Omega_{t-1}, \Delta y_t] \\
&= \frac{f(S_t = j, S_{t-1} = i, \Delta y_t | \Omega_{t-1})}{f(\Delta y_t | \Omega_{t-1})} \\
&= \frac{f(\Delta y_t | S_t = j, S_{t-1} = i, \Omega_{t-1}) \Pr[S_t = j | S_{t-1} = i | \Omega_{t-1}]}{f(\Delta y_t | \Omega_{t-1})}
\end{aligned}$$

with

$$\Pr[S_t = j | \Omega_t] = \sum_{i=1}^4 \Pr[S_t = j, S_{t-1} = i | \Omega_t]$$

A2 Constructing T_t^w from $\Delta \tau_t^w$

Since the model is estimated in deviations from means, δ is concentrated out of the likelihood function. Following Stock and Watson (1991), the steady state Kalman gain can be used to retrieve this term.

$$\delta = E'(I_n - (I_n - K^*H)F)^{-1} K^* \Delta \bar{y}$$

where K^* is the steady state Kalman gain, $E' = [1 \ 0 \ 0 \ \dots \ 0]$, and n is the dimension of the state vector. Once δ is obtained, given $\Delta \tilde{\tau}_T^w = [\Delta \tau_1^w \ \Delta \tau_2^w \ \dots \ \Delta \tau_T^w]$ and arbitrary initial value T_0^w , T_t^w is constructed as:

$$T_t^w = \delta + \Delta \tau_t^w + T_{t-1}^w,$$

where $t = 1, 2, \dots, T$.

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